METR 3113 | Quiz #3 | November 19, 2007

Instructions: NO CALCULATORS. Use the distributed BLUE BOOK for submitting your answers. Print your name, and sign your legal signature on the front of the blue book. Print your name on the back of the blue book. Answer 1 or 2, 3 or 4, 5 or 6, 7 or 8. 10 pts each. INDICATE ON THE FRONT OF THE BLUE BOOK WHICH 4 QUESTIONS YOU WANT TO BE GRADED.

1. **f-plane Choice 1.**

   (a) (5 pts) From

   $$\frac{dv_\theta}{dt} + \frac{v_\theta v_r}{r} = -f v_r - g_o \frac{1}{r}\frac{\partial Z}{\partial \theta}$$

   derive

   $$r v_\theta + \frac{1}{2} f r^2 = \text{const.}$$

   What assumptions have been made?

   (b) (5 pts) Consider a ring of air with initial radius $r = 200$ km and $v_\theta = 0$. The ring is at low latitudes where $f = 0.5 \times 10^{-4}\ \text{s}^{-1}$. The ring contracts to a radius of $r = 100$ km. What is the resulting value of v_θ?

2. **f-plane Choice 2.** Consider

   $$\frac{d\vec{V}}{dt} = \vec{V} \times f\widehat{\mathbf{k}} - \frac{1}{\rho}\nabla_H p \qquad (1)$$

   (a) (5 pts) Derive the energy equation from the horizontal equation of motion: What conclusions can you make from the energy equation?

   (b) (5 pts) With $\vec{V} = u\widehat{\mathbf{\imath}} + v\widehat{\mathbf{\jmath}}$, show the $\widehat{\mathbf{\imath}}$ and $\widehat{\mathbf{\jmath}}$ momentum equation components ($\frac{du}{dt} = ..., \frac{dv}{dt} = ...$). What simplifying approximations did you make? Why are they justified?

3. **Tidal Force Choice 1.** Suppose the Hand of God grinds up the moon and fashions a new moon that is placed at 1/3 the current distance from Earth. As seen in the sky, the new moon is the same angular diameter as currently seen by us. The Hand of God discards the excess lunar material.

   (a) (3 pts) What is the ratio of the period of orbit of the new moon to the old moon?

   (b) (3 pts) What is the ratio of the force of the gravity of the new moon, upon the Earth, as compared to that of the old moon?

   (c) (4 pts) What is the ratio of the tidal force the new moon, upon the Earth, as compared to that of the old moon?

4. **Tidal Force Choice 2.**

   (a) (5 pts) Determine the ratio of the average density of the sun to the average density of the moon based on knowledge of the ratio of the magnitudes of the maximum tidal force and the fact that the sun and moon appear to be the same size in the sky.

(b) (5 pts) Now consider only the Earth-Moon system. Make a rough sketch of the tidal force vectors on the surface of the Earth, as caused by the Moon.

5. **Gradient Wind Choice 1.** Consider a planet just like Earth, except the acceleration due to gravity is exactly $g = 10$ m s $^{-2}$. Suppose at a certain instant in time, the velocity vector of a parcel of air is directed exactly toward north, (the local y direction), with a meridional velocity component $v = 10$ m s^{-1}. Suppose the Coriolis parameter is exactly $f = 10^{-4}$ s^{-1}. In the following questions, Z is the height of the pressure surface in which the parcel lies, and x is the coordinate directed toward the east.

   (a) (2 pts) Suppose the trajectory of the parcel is straight. Find $\frac{\partial Z}{\partial x}$.

   (b) (2 pts) Suppose the trajectory is curved toward the west, with a radius of curvature of 2000 km. Find $\frac{\partial Z}{\partial x}$.

   (c) (2 pts) Suppose the trajectory is curved toward the east, with a radius of curvature of 2000 km. Find $\frac{\partial Z}{\partial x}$.

   (d) (2 pts) Suppose the trajectory is curved toward the west, with a radius of curvature of 100 m. (Think tornado). Find $\frac{\partial Z}{\partial x}$.

   (e) (2 pts) In (d), suppose the density is $\rho = 1$ kg m^{-3}. Find $\frac{\partial p}{\partial x}$ where p is pressure. Express the answer with units of millibar per meter.

6. **Gradient Wind Choice 2.** The vector equation of motion for the mythical frictionless hockey puck on a horizontal surface, or an air parcel with no horizontal pressure gradient forces, is:

$$\frac{d\vec{V}}{dt} = f\vec{V} \times \widehat{\mathbf{k}} \tag{2}$$

Suppose you choose to analyze (2) in Cartesian coordinates, with $\vec{V} = u\widehat{\mathbf{\imath}} + v\widehat{\mathbf{\jmath}}$. Eq. (2) is:

$$\frac{du}{dt} = fv \tag{3}$$

$$\frac{dv}{dt} = -fu \tag{4}$$

$$\tag{5}$$

   (a) (2 pts) Show that

$$\frac{d}{dt}\left(u^2 + v^2\right) = 0 \tag{6}$$

   Thus $u^2 + v^2 =$ const.

   (b) (2 pts) Suppose $v(0) = V$ and and $u(0) = 0$. Thus $u^2 + v^2 = V^2$. $\frac{du}{dt} = fv$ can be written

$$\frac{du}{dt} = f\sqrt{V^2 - u^2} \tag{7}$$

   Show that $u(t) = V\sin(ft)$ is a solution to (7).

   (c) (2 pts) With $u(t)$ now known, find $v(t)$.

(d) (2 pts) Suppose $x(0) = 0$ and $y(0) = 0$. Given that $\frac{dx}{dt} = u$ and $\frac{dy}{dt} = v$, find $x(t)$ and $y(t)$.

(e) (2 pts) In terms of V and f, what is the maximum distance the hockey puck (air parcel) strays from the origin? Is this answer consistent with what you deduce from the polar coordinate solution?

7. **Continuum analysis of flows. Choice 1.** Consider a velocity vector field in a plane: $\vec{U} = u\widehat{\mathbf{\imath}} + v\widehat{\mathbf{\jmath}}$ with $u = \gamma x$ and $v = -\gamma y$, where γ is a constant, with dimensions of inverse time.

   (a) (1 pts) Sketch the velocity field in the quadrant $x \geq 0$ and $y \geq 0$. Assume $\gamma > 0$.

   (b) (2 pts) Calculate $\nabla \cdot \vec{U}$. Is this flow incompressible?

   (c) (3 pts) Find equations for the curves that are the streamlines.

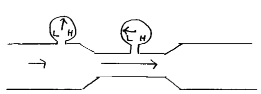
   (d) (4 pts) Find the pressure field $p(x, y)$ given that:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

   Assume density is constant.

8. **Continuum analysis of flows. Choice 2.** (10 pts)



Derive how measurement of both static pressure on the interior wall of a constriction in a pipe and the static pressure on the interior wall of the pipe, away from the constriction, can be used to diagnose the rate of flow (volume or mass) through the pipe.