METR 3113: Atmospheric Dynamics 1

Buoyancy

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Lecture for Friday, September 28, 2007

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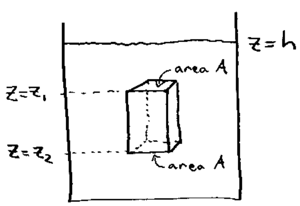

A solid cuboid submerged in a static fluid of constant density ρ_{F}. The object is tethered to the bottom with a thread. In the fluid, *even underneath the object*,

$$p(z) = \rho_{\mathrm{F}} g(h - z) + p_a$$

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On the top face:

$$p_1 = \rho_{\mathrm{F}} g(h - z_1) + p_a$$

On the bottom face:

$$p_2 = \rho_{\mathrm{F}} g(h - z_2) + p_a$$

The net upward force of pressure is

$$\begin{aligned} F_B &= A\,p_2 - A\,p_1 = A(p_2 - p_1) = A\rho_{\mathrm{F}} g(z_1 - z_2) \\ &= g\rho_{\mathrm{F}} V \qquad \text{because } V = A(z_1 - z_2) \\ &= g m_{\mathrm{F}} \qquad m_{\mathrm{F}} \text{ is mass of fluid displaced by object} \end{aligned}$$

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$$F_B = g m_{\mathrm{F}}$$

This net upward thrust of pressure F_B is the *engineering buoyancy force*.

The above is *Archimedes' Principle*.

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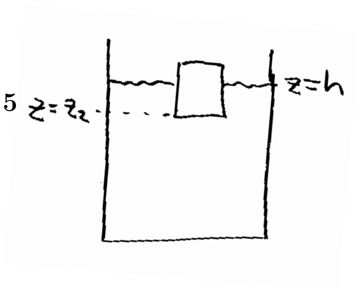

An object floating in the fluid. Now $p_1 = p_a$.

$$F_B = A(p_2 - p_1) = A\rho_{\text{F}} g(h - z_2)$$

$$F_B = g\rho_{\text{F}} V = m_{\text{F}}$$

V is the volume *displaced* by the fluid, NOT the volume of the object.

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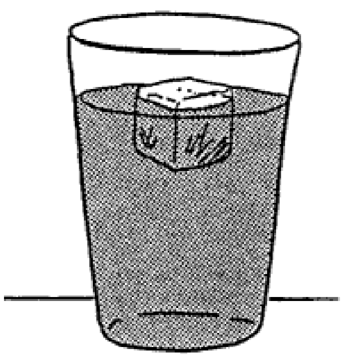

The weight of the ice cube $m_{\text{ice}}g$ is balanced by the buoyancy force:

$$m_{\text{ice}}g = F_B = m_{\text{F}}g$$

The mass of water displaced is exactly equal to the mass of the ice:

$$m_{\text{ice}} = m_{\text{F}}$$

The melted ice fits exactly into the "hole" of displaced water. The water level does not rise.

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The *meteorology buoyancy force* F_b is often the upward force of pressure minus the weight due to the mass m_{o} of the object:

$$F_b = F_B - gm_{\text{o}} = g(m_{\text{F}} - m_{\text{o}})$$

With vertical acceleration given by:

$$m_{\text{o}}a = F_b = g(m_{\text{F}} - m_{\text{o}})$$

$$a = g\frac{m_{\text{F}} - m_{\text{o}}}{m_{\text{o}}}$$

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Buoyancy in arbitrary blobs of fluid:

Vertical acceleration :

$$\frac{dw}{dt} = -g - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\rho\frac{dw}{dt} = -\rho g + \left(-\frac{\partial p}{\partial z}\right)$$

On the right, the first term is always negative, and the second term *almost* always positive. (The exception being locally extreme circumstances, such as in a suction vortex in a tornado.) When the first term wins we might say there is *negative buoyancy*. When the second term wins we might say there is *positive buoyancy*.

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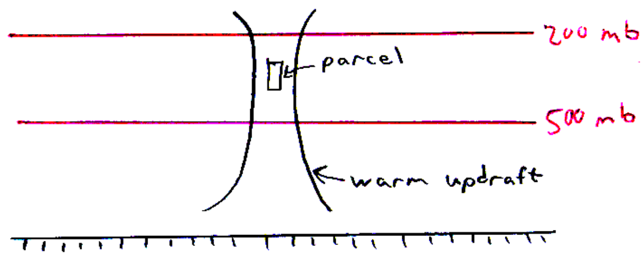

Consider a blob (or parcel) embedded in a narrow updraft of a thunderstorm.

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Pressure and density outside the updraft is denoted with an overbar. Outside the updraft, hydrostatic balance occurs:

$$\overline{\rho}\frac{dw}{dt} = -\overline{\rho}g - \frac{d\overline{p}}{dz} = 0$$

or

$$\frac{d\overline{p}}{dz} = -\overline{\rho}g$$

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Inside the updraft:

$$\rho\frac{dw}{dt} = -\rho g - \frac{dp}{dz}$$

Now the key approximation: If the updraft is sufficiently narrow, the external pressure field (at the same elevation) imposes itself on the updraft:

$$\frac{dp}{dz} = \frac{d\overline{p}}{dz}$$

$$\rho\frac{dw}{dt} = -\rho g - \frac{d\overline{p}}{dz}$$

$$\rho\frac{dw}{dt} = -\rho g + \overline{\rho}g$$

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$$\frac{dw}{dt} = g\frac{\overline{\rho} - \rho}{\rho}$$

The right hand side is often call *the buoyancy* or *the buoyant force*. Compare with what we saw earlier for the vertical acceleration of an object in a fluid:

$$a = g\frac{m_{\text{F}} - m_{\text{O}}}{m_{\text{O}}}$$

$$\frac{dw}{dt} = g\frac{\overline{\rho} - \rho}{\rho}$$

Can also be expressed in terms of temperature differences:

$$p = R\rho T \qquad \rho = \frac{p}{RT} \qquad \overline{\rho} = \frac{\overline{p}}{R\overline{T}}$$

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$$\frac{dw}{dt} = g\frac{\frac{\overline{p}}{R\overline{T}} - \frac{p}{RT}}{\frac{p}{RT}}$$

But $\overline{p} = p$ so:

$$\frac{dw}{dt} = g\frac{T - \overline{T}}{\overline{T}}$$

$$\frac{dw}{dt} = g\frac{T - \overline{T}}{\overline{T}}$$

can be expressed in terms of differences in *potential temperature* Θ:

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$$T = \Theta\left(\frac{p}{p_o}\right)^{R/c_p}$$

Again using $p = \overline{p}$,

$$\frac{dw}{dt} = g\frac{\Theta - \overline{\Theta}}{\overline{\Theta}}$$