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METR 3113: Atmospheric Dynamics 1

Draft: The Coriolis Force for Meteorology

Lecture for Wednesday, October 17, 2007

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We have derived the Coriolis force and centrifugal force for acceleration of wind relative to a coordinate system and unit vectors fixed on Earth:

$$\frac{d\vec{U}}{dt} = \vec{U} \times 2\vec{\Omega} + \vec{g}_e - \frac{1}{\rho}\nabla p$$

The centrifugal force is contained within the definition of *effective gravity*:

$$\vec{g}_e \equiv \vec{g} + \Omega^2 \vec{R}$$

We next resolve the Coriolis force (per unit mass) $\vec{U} \times 2\vec{\Omega}$ into a form useful for meteorology.

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The velocity vector is written

$$\vec{U} = u\widehat{\mathbf{i}} + v\widehat{\mathbf{j}} + w\widehat{\mathbf{k}}$$

The movement of the unit vectors relative to the "fixed stars" has already been accounted for in the Coriolis force and centrifugal force terms that were derived.

Note: if a local polar coordinate system is applied:

$$\vec{U} = v_r\widehat{\mathbf{r}} + v_\theta\widehat{\boldsymbol{\theta}} + w\widehat{\mathbf{k}}$$

then $\frac{d\vec{U}}{dt}$ will need to account for the movement of $\widehat{\mathbf{r}}$ and $\widehat{\boldsymbol{\theta}}$ relative to $\widehat{\mathbf{i}}$ and $\widehat{\mathbf{j}}$.

An additional centripetal acceleration term will appear.

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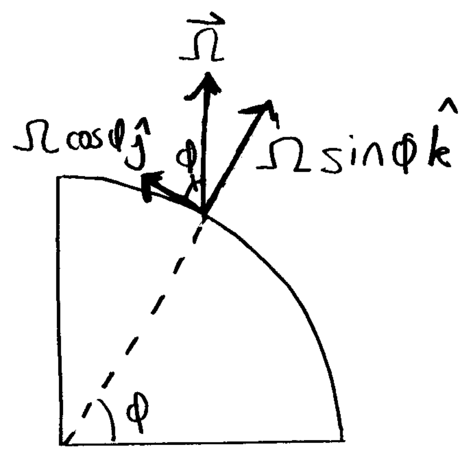

The rotation vector Ω, resolved into the components of a local Cartesian coordinate system. ϕ is latitude.

$$\vec{\Omega} = \Omega \sin\phi\ \widehat{\mathbf{k}} + \Omega\cos\phi\ \widehat{\mathbf{j}}$$

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$$\begin{aligned}\vec{U}\times 2\vec{\Omega} &= 2\Omega\left(u\widehat{\mathbf{\imath}}+v\widehat{\mathbf{\jmath}}+w\widehat{\mathbf{k}}\right)\times\left(\sin\phi\widehat{\mathbf{k}}+\cos\phi\widehat{\mathbf{\jmath}}\right)\\ &= 2\Omega\left(u\sin\phi\widehat{\mathbf{\imath}}\times\widehat{\mathbf{k}}+u\cos\phi\widehat{\mathbf{\imath}}\times\widehat{\mathbf{\jmath}}\right.\\ &\quad\left.+v\sin\phi\widehat{\mathbf{\jmath}}\times\widehat{\mathbf{k}}+w\cos\phi\widehat{\mathbf{k}}\times\widehat{\mathbf{\jmath}}\right)\\ &= 2\Omega\left(-u\sin\phi\widehat{\mathbf{\jmath}}+u\cos\phi\widehat{\mathbf{k}}+v\sin\phi\widehat{\mathbf{\imath}}-w\cos\phi\widehat{\mathbf{\imath}}\right)\\ &= 2\Omega\sin\phi\left(v\widehat{\mathbf{\imath}}-u\widehat{\mathbf{\jmath}}\right)+2\Omega\cos\phi\left(u\widehat{\mathbf{k}}-w\widehat{\mathbf{\imath}}\right)\end{aligned}$$

Both of the terms proportional to $2\Omega\cos\phi$ are not significant in meteorology: The term with $u\widehat{\mathbf{k}}$ is in the vertical, and very small compared with gravity and the pressure gradient force. The term proportional to w is small compared to the other terms proportional to either u or v, because w is small compared to u and v.

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$$\begin{aligned}\vec{U}\times 2\vec{\Omega} &= 2\Omega\sin\phi\left(v\widehat{\mathbf{\imath}}-u\widehat{\mathbf{\jmath}}\right)+2\Omega\cos\phi\left(u\widehat{\mathbf{k}}-w\widehat{\mathbf{\imath}}\right)\\ &\approx 2\Omega\sin\phi\left(v\widehat{\mathbf{\imath}}-u\widehat{\mathbf{\jmath}}\right)\end{aligned}$$

Continuing with the approximate form,

$$\begin{aligned}\vec{U}\times 2\vec{\Omega} &= 2\Omega\sin\phi\left(v\widehat{\mathbf{\imath}}-u\widehat{\mathbf{\jmath}}\right)\\ &= 2\Omega\sin\phi\left(u\widehat{\mathbf{\imath}}+v\widehat{\mathbf{\jmath}}\right)\times\widehat{\mathbf{k}}\\ &= \vec{V}\times f\widehat{\mathbf{k}}\end{aligned}$$

where $\vec{V}$ is the horizontal wind vector:

$$\vec{V}\equiv u\widehat{\mathbf{\imath}}+v\widehat{\mathbf{\jmath}}$$

and f is the *Coriolis parameter*:

$$f\equiv 2\Omega\sin\phi$$

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The length of day relative to the stars is a *sidereal day* or 23 hours, 56 minutes, 4.1 seconds .

$$\Omega=\frac{2\pi}{86164\ \mathrm{s}}=7.292\times 10^{-5}\ \mathrm{s}^{-1}$$

$$f=2\sin(\phi)\ \ \Omega\approx 10^{-4}\ \mathrm{s}^{-1}$$

$$|\vec{V}\times f\widehat{\mathbf{k}}|=fV$$

A typical value of $V=10\ \mathrm{m\ s}^{-1}$, so a typical value of the Coriolis acceleration is $fV=10^{-3}\ \mathrm{m\ s}^{-2}$

Recall that a typical value of $|\frac{1}{\rho}\frac{\partial p}{\partial x}|$ is

$$\left|\frac{1}{\rho}\frac{\partial p}{\partial x}\right|=\frac{1}{1\ \mathrm{kg\ m}^{-3}}\frac{1000\ \mathrm{Pa}}{1000\ \mathrm{km}}=10^{-3}\ \mathrm{m\ s}^{-2}$$

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$$\vec{V}\times f\widehat{\mathbf{k}}$$

- Proportional to horizontal windspeed V.
- Acts in the horizontal, to right of the horizontal wind vector in the NH, to the left in the SH.
- Increases in magnitude toward the poles.
- Typical magnitude is $10^{-3}\ \mathrm{m\ s}^{-2}$, the same order of magnitude as the horizontal pressure gradient force.

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Assume a car is travelling toward the east on Lindsey St. with speed $V = 10\ \mathrm{m\ s^{-1}}$. Assume the car is travelling over an icy, frictionless section of the road that is exactly flat. (This means if the car was stationary, if could remain in equilibrium on the road).

After $10\ \mathrm{s}$, by how much is the car deflected toward the south? Submit the answer in units of centimeters.

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$$f = 2\Omega \sin(35^\circ) = 0.000083\ \mathrm{s^{-1}}$$

$a = fV = 0.000083\,\mathrm{s^{-1}}\ 10\,\mathrm{m\,s^{-1}} = 0.00083\,\mathrm{m\,s^{-2}}$

The deflection is

$$d = \frac{1}{2}at^2 = 0.0415\ \mathrm{m} = 4.15\ \mathrm{cm}$$

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Suppose instead the road slopes downward to the left with an angle θ relative to horizontal. What angle θ would allow the car to go straight, undeflected by the Coriolis force?

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$$g \sin(\theta) = fV \cos(\theta)$$

For small θ,

$$\theta = \frac{fV}{g} = 8.46 \times 10^{-5} \approx 10^{-4}$$