METR 5223: Atmospheric Radiation

1:

Elementary Adjustment to Radiative Equilibrium Part 2

Lecture for Spring 2009
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2:

$$\tau \frac{dT'}{dt} + T' = T'_0$$

For notational simplicity, let:

$$x \equiv \frac{t}{\tau} \qquad y \equiv T' \qquad f \equiv T'_0$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y = f(x)$$

3:

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y = f(x)$$

$$e^x \frac{\mathrm{d}y}{\mathrm{d}x} + e^x y = e^x f(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^x y\right) = e^x f(x)$$

$$e^x y\big|_{x_0}^{x} = \int_{x_0}^{x} f(x') e^{x'} \mathrm{d}x'$$

$$e^x y(x) - e^{x_0} y(x_0) = \int_{x_0}^{x} f(x') e^{x'} \mathrm{d}x'$$

4:

$$y(x) = y(x_0) e^{x_0 - x} + \int_{x_0}^{x} f(x') e^{x' - x} \mathrm{d}x'$$

Define a **propagator**:

$$g(x', x) \equiv e^{x' - x}$$

Or, since $x > x'$, and to emphasize $g(x', x)$ represents exponential decay,

$$g(x', x) = e^{-(x - x')}$$

$$y(x) = y(x_0) g(x_0, x) + \int_{x_0}^{x} f(x') g(x', x) \mathrm{d}x'$$

5:

Note that x' is "more variable" than x.

Meaning when seeking a solution for y at x, x' is the "variable variable".

$$g(x', x) \equiv e^{x'-x}$$

$$\mathrm{d}g(x', x) = \mathrm{d}e^{x'-x} = e^{x'-x}\mathrm{d}x' = g(x', x)\mathrm{d}x'$$

6:

So, because the propagator is exponential,

$$g(x', x)\mathrm{d}x' = \mathrm{d}g(x', x)$$

the solution converts to:

$$y(x) = y(x_0)g(x_0, x) + \int_{x_0}^{x} f(x')g(x', x)\mathrm{d}x'$$

$$y(x) = y(x_0)g(x_0, x) + \int_{x_0}^{x} f(x')\mathrm{d}g(x', x)$$

$$y(x) = y(x_0)g(x_0, x) + \int_{g(x_0,x)}^{g(x,x)} f(x')\mathrm{d}g(x', x)$$

$$y(x) = y(x_0)g(x_0, x) + \int_{g(x_0,x)}^{1} f(x')\mathrm{d}g(x', x)$$

7:

Consider a $f(x)$ that is represent as piece-wise constant:

$$f(x) = f_1, \; x_0 < x < x_1$$

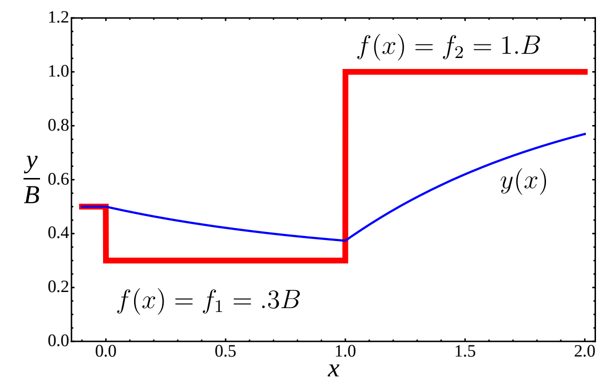
$$f(x) = f_2, \; x_1 < x < x_2$$

and so on. When solving for x in the second layer, or $x_1 < x < x_2$:

$$y(x) = y(x_0)g(x_0, x) + \int_{g(x_0,x)}^{g(x_1,x)} f_1 \mathrm{d}g(x', x) + \int_{g(x_1,x)}^{1} f_2 \mathrm{d}g(x', x)$$

$$y(x) = y(x_0)g(x_0, x) + f_1\left[g(x_1, x) - g(x_0, x)\right] + f_2\left[1 - g(x_1, x)\right]$$

8:



$y(x)$ for $y(0) = .5B$ and the depicted $f(x)$.

Reality check.

Suppose $f_1 = f_2 = y(x_0)$:

9:

$$
\begin{aligned}
y(x) = \quad & y(x_0)g(x_0, x) \\
+ \quad & y(x_0)\,[g(x_1, x) - g(x_0, x)] \\
+ \quad & y(x_0)\,[1 - g(x_1, x)] \\
= \quad & y(x_0)
\end{aligned}
$$

Reality checks will help prevent certain common errors.

Here is how an error develops. This is **okay**:

$$
\begin{aligned}
y(x_2) = \quad & y(x_0)g(x_0, x_2) \\
+ \quad & f_1\,[g(x_1, x) - g(x_0, x_2)] \\
+ \quad & f_2\,[1 - g(x_1, x_2)]
\end{aligned}
$$

10: Adding another layer and simply recycling your previous computations is **not okay**:

$$
\begin{aligned}
y(x_3) = \quad & y(x_0)g(x_0, x_2) \\
+ \quad & f_1\,[g(x_1, x) - g(x_0, x_2)] \\
+ \quad & f_2\,[1 - g(x_1, x_2)] \\
+ \quad & f_3\,[1 - g(x_2, x_3)]
\end{aligned}
$$