METR 5223: Atmospheric Radiation

Radiative Equilibrium in a Gray Atmosphere

Lecture for Spring 2009, v0.3

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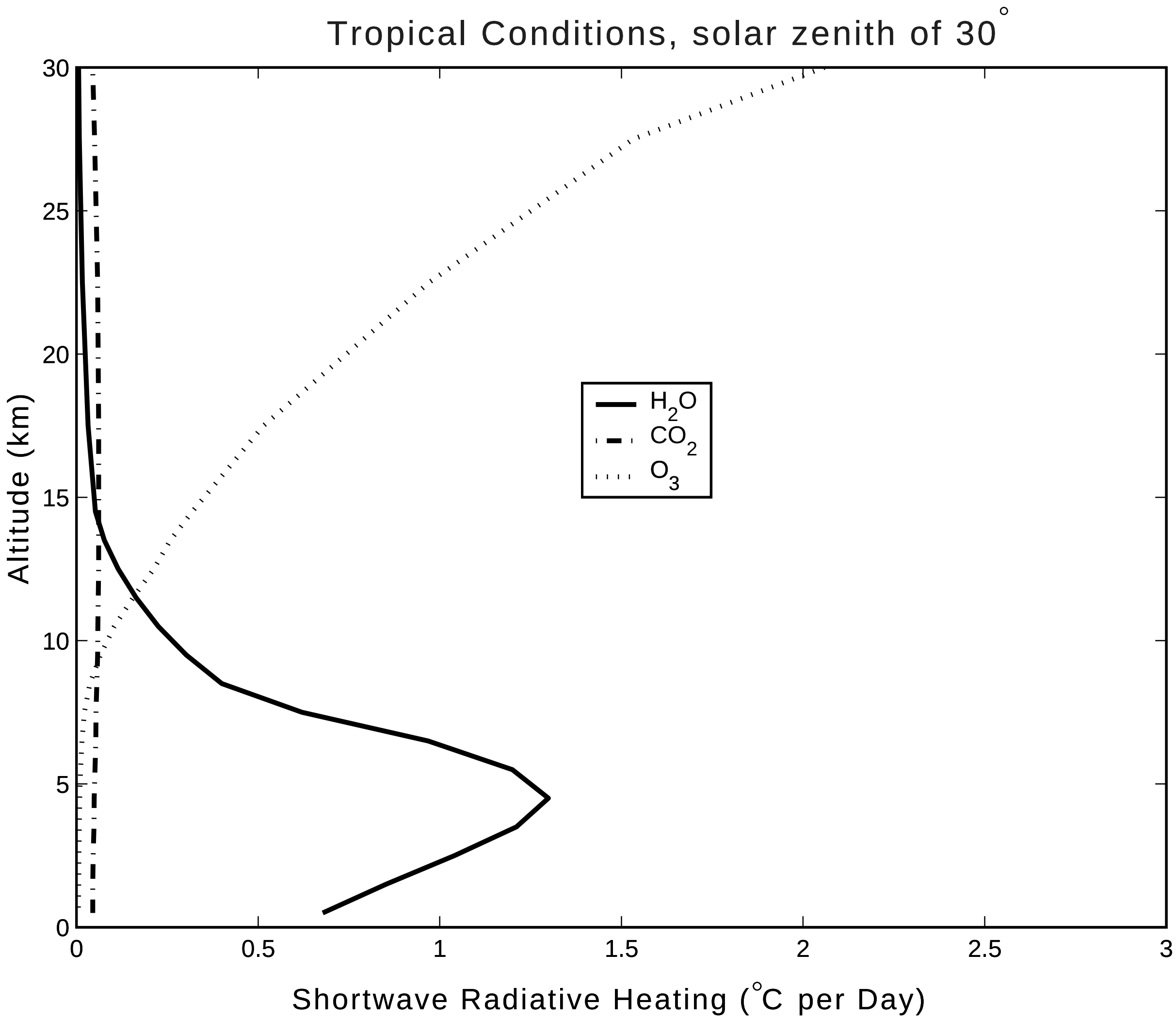
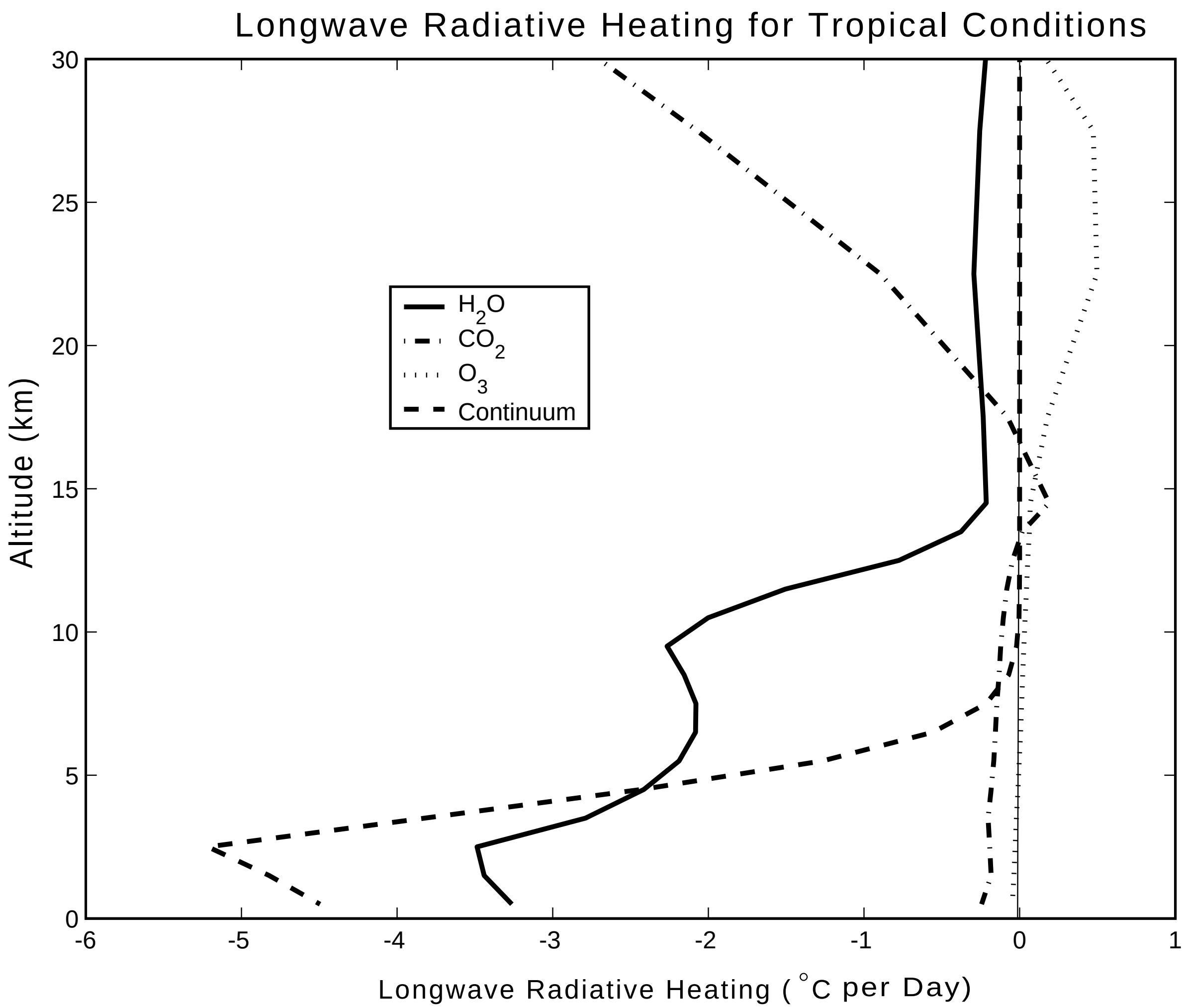


Typical heating profiles due to solar absorption in a cloud-free tropical atmosphere.



Typical heating profiles due to longwave radiative transfer in a cloud-free tropical atmosphere.

Idealized radiative equilibrium solutions

- Simplified with all IR having the same value of k_ν.
- No sensible or latent heat fluxes.
- Steady solar heating at surface, and optionally in the atmosphere.
- Simple analytical solution of ODEs.
- Demonstrates stratospheric cooling associated with increased greenhouse gases.

Optical depth at a certain height z is the optical path (thickness) from z to the top of the atmosphere:

$$\chi_\nu(z) = \int_z^\infty k_\nu(z')\rho_a(z')dz'$$

For application to irradiance we use $\chi_\nu^* = 1.66\chi_\nu$ and

$$\frac{dF_\nu \downarrow}{d\chi_\nu^*} = -F_\nu \downarrow + \pi B_\nu$$

Leave the * implied:

$$\frac{dF_\nu\downarrow}{d\chi_\nu} + F_\nu\downarrow = \pi B_\nu$$

Also, in a gray atmosphere χ_ν and k_ν are independent of ν, so $\chi_\nu = \chi$. Integrate over ν:

$$\frac{dF\downarrow}{d\chi} + F\downarrow = \sigma T^4$$

Similarly:

$$-\frac{dF\uparrow}{d\chi} + F\uparrow = \sigma T^4$$

$F\downarrow$, $F\uparrow$ and σT^4 are coupled. Need another equation.

In radiative equilibrium, the divergence of the net upward flux is zero:

$$\rho c_p \frac{dT}{dt} = -\frac{d}{dz}(F \uparrow - F \downarrow) = 0$$

So

$$F \uparrow - F \downarrow = F_\circ$$

where $F_\circ$ is a constant. At $\chi = 0$ (T.O.A), $F \downarrow (0) = 0$ and $F \uparrow (0) = F_\circ$. Thus $F_\circ$ is also the net downward solar irradiance at the T.O.A..

Recall:

$$\frac{dF\downarrow}{d\chi} + F\downarrow = \sigma T^4$$

$$-\frac{dF\uparrow}{d\chi} + F\uparrow = \sigma T^4$$

Adding the previous 2 Schwarzschild equations together:

$$\frac{d}{d\chi}(F\downarrow - F\uparrow) + F\downarrow + F\uparrow = 2\sigma T^4$$

But

$$\frac{d}{d\chi}(F\downarrow - F\uparrow) = 0$$

So

$$F\downarrow + F\uparrow = 2\sigma T^4$$

$$F\downarrow + F\uparrow = 2\sigma T^4$$

If we multiply both sides by absorptance/emittance:

$$\text{absorption} = \text{emission}$$

Absorption is proportional to the irradiances passing through a layer.

Emission is proportional to $2\sigma T^4$ (the layer radiates in both directions).

Combine

$$F \downarrow + F \uparrow = 2\sigma T^4$$

and

$$F \uparrow - F \downarrow = F_\circ$$

to give

$$F \uparrow - F_\circ + F \uparrow = 2\sigma T^4$$

or

$$F \uparrow = \sigma T^4 + \frac{1}{2} F_\circ$$

and

$$F \downarrow = \sigma T^4 - \frac{1}{2} F_\circ$$

Rewrite previous equation as:

$$\sigma T^4 = F\downarrow + \frac{1}{2} F_\circ$$

Substitute into

$$\frac{dF\downarrow}{d\chi} + F\downarrow = \sigma T^4$$

$$\frac{dF\downarrow}{d\chi} + F\downarrow = F\downarrow + \frac{1}{2} F_\circ$$

Solution is simple. Using $F\downarrow(0) = 0$,

$$F\downarrow(\chi) = \frac{1}{2} F_\circ \chi$$

Recap:

$$F\downarrow(\chi) = \frac{1}{2}F_\circ\chi$$

$$\sigma T^4(\chi) = \frac{1}{2}F_\circ\chi + \frac{1}{2}F_\circ$$

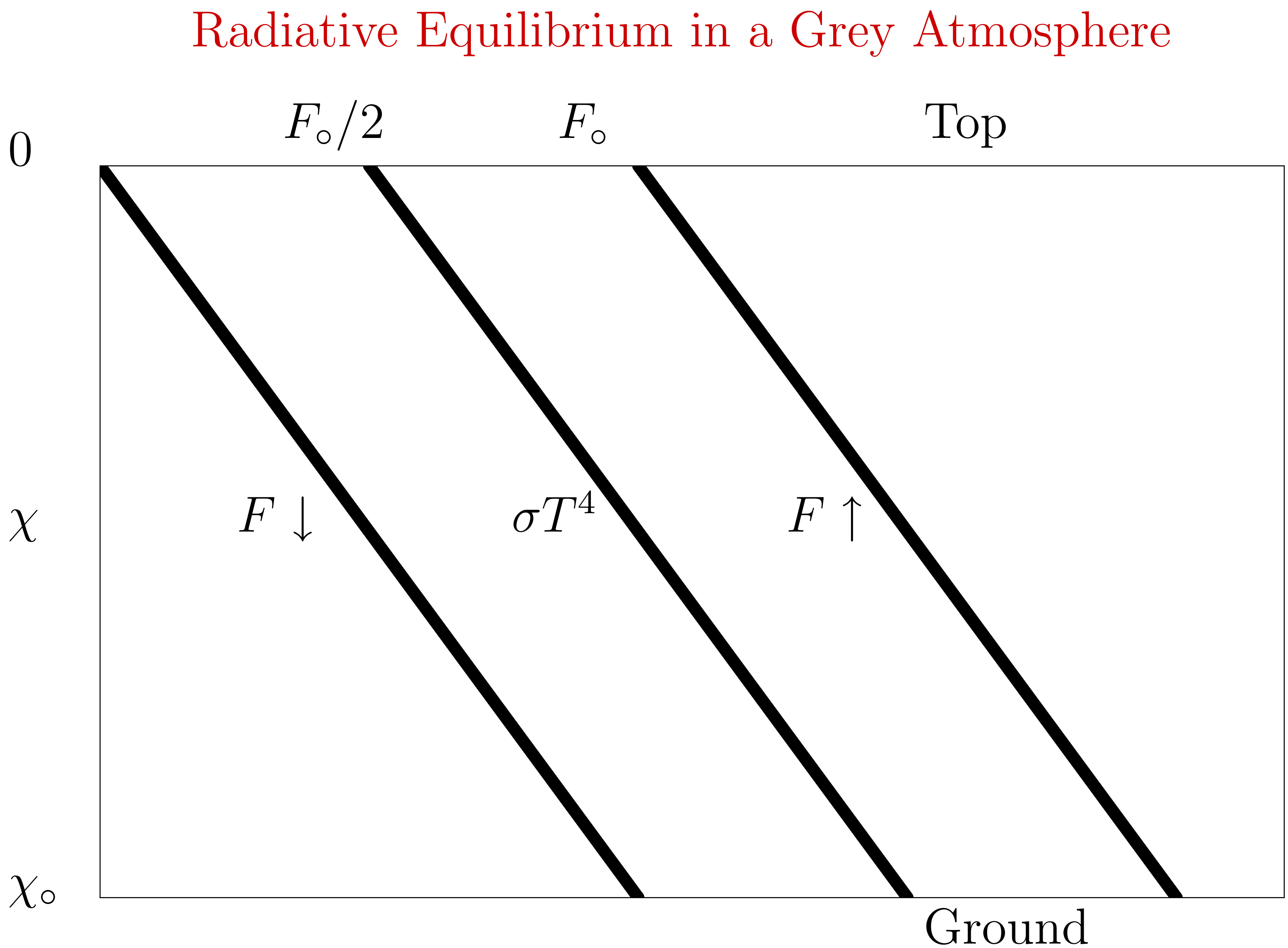
$$F\uparrow(\chi) = \frac{1}{2}F_\circ\chi + F_\circ$$

Note at the ground, or $\chi = \chi_\circ$, T_g is given by:

$$F\uparrow(\chi_\circ) = \frac{1}{2}F_\circ\chi_\circ + F_\circ = \sigma T_g^4$$

But the air temperature at the ground is T_a:

$$\sigma T_a^4 = \sigma T_g^4 - \frac{1}{2}F_\circ$$


Radiative Equilibrium in a Grey Atmosphere
$F_\circ/2$
$F_\circ$
Top
0
χ
$F\downarrow$
σT^4
$F\uparrow$
$\chi_\circ$
Ground

Generalize the gray atmosphere model to include a prescribed solar absorption acting on a downward solar irradiance Q.

Model is more clear using dimensionless pressure p for the independent variable. $p = 0$ at the top of the atmosphere and $p = 1$ at the surface.

The optical path (optical depth) for longwave radiation is now simply $\chi = \chi_{\circ} p$.

$$Q = F_{\circ} - sF_{\circ}\left(1 - e^{-\alpha p}\right)$$

α controls the rate of reduction with p.

s controls the fraction of Q that is ultimately removed.

We do experiments for increasing $\chi_\circ$ (the value of χ at the surface) to explore an increased greenhouse effect.

We will hold α and s invariant, which keeps solar absorption invariant in its magnitude and position.

The solar irradiance Q is positive downward.

The solar heating is proportional to the convergence of the solar irradiance, or

$$-\frac{\mathrm{d}Q}{\mathrm{d}p}$$

The longwave heating is proportional to the convergence of the net downward irradiance,

$$-\frac{\mathrm{d}}{\mathrm{d}p}\left(F\downarrow - F\uparrow\right)$$

There is no heating by latent heat release or convection in the gray atmosphere model. The sum of the radational heating is zero:

$$-\frac{\mathrm{d}}{\mathrm{d}p}\left(Q + F\downarrow - F\uparrow\right) = 0$$

In equilibrium, there is 0 net irradiance at the top of the atmosphere. So the above equation integrates to

$$Q + F\downarrow - F\uparrow = 0$$

for all p.

Recall the Schwarzschild equations for the downward and upward longwave irradiance:

$$\frac{1}{\chi_\circ}\frac{\mathrm{d}F\downarrow}{\mathrm{d}p} + F\downarrow = \sigma T^4$$

$$-\frac{1}{\chi_\circ}\frac{\mathrm{d}F\uparrow}{\mathrm{d}p} + F\uparrow = \sigma T^4$$

Recall the goal here is to solve for $F\uparrow$, $F\downarrow$ and σT^4 using the boundary conditions $F\uparrow(0) = F_\circ$ and $F\downarrow(0) = 0$, and the zero net heating condition:

$$\frac{\mathrm{d}}{\mathrm{d}p}\left(Q + F\downarrow - F\uparrow\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}p}\left(Q + F\downarrow - F\uparrow\right) = 0$$

can be written

$$\frac{\mathrm{d}Q}{\mathrm{d}p} = \frac{\mathrm{d}F\uparrow}{\mathrm{d}p} - \frac{\mathrm{d}F\downarrow}{\mathrm{d}p}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}p} = \chi_\circ\left(F\downarrow + F\uparrow - 2\sigma T^4\right)$$

$$\frac{\mathrm{d}Q}{\mathrm{d}p} = \chi_\circ\left(2F\downarrow + Q - 2\sigma T^4\right)$$

Using

$$\frac{\mathrm{d}Q}{\mathrm{d}p} = \chi_\circ \left(2F\downarrow + Q - 2\sigma T^4\right)$$

to eliminate σT^4 from

$$\frac{1}{\chi_\circ}\frac{\mathrm{d}F\downarrow}{\mathrm{d}p} + F\downarrow = \sigma T^4$$

gives

$$\frac{1}{\chi_\circ}\frac{\mathrm{d}}{\mathrm{d}p}\left(F\downarrow + \frac{1}{2}Q\right) = \frac{1}{2}Q$$

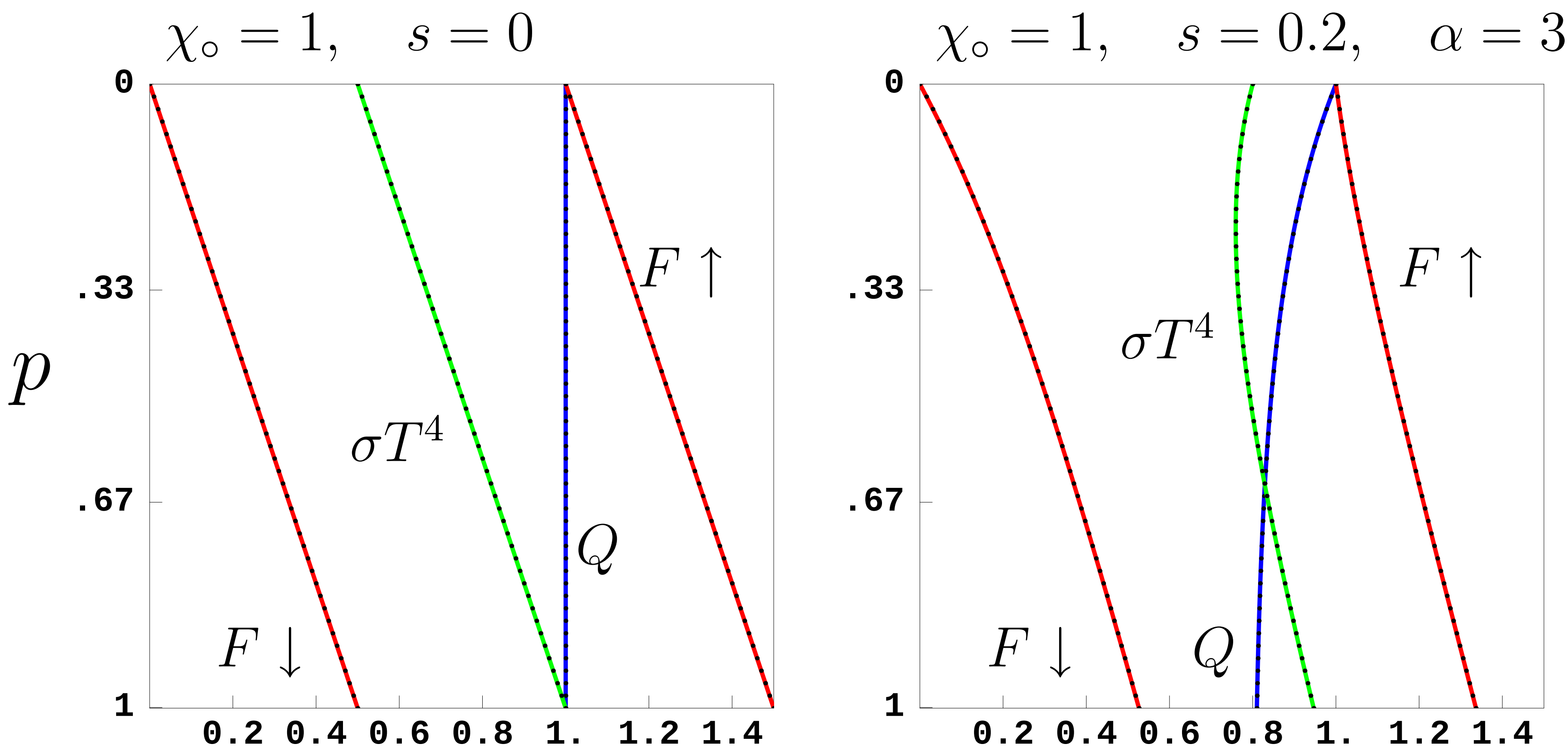
This has solution

$$F\downarrow = \frac{1}{2}F_\circ(1-s)\chi_\circ p + \frac{1}{2}F_\circ s\left(1 + \frac{\chi_\circ}{\alpha}\right)\left(1 - e^{-\alpha p}\right)$$

Using the solution for $F\downarrow(p)$, we obtain $\sigma T^4(p)$ and $F\uparrow(p)$ using:

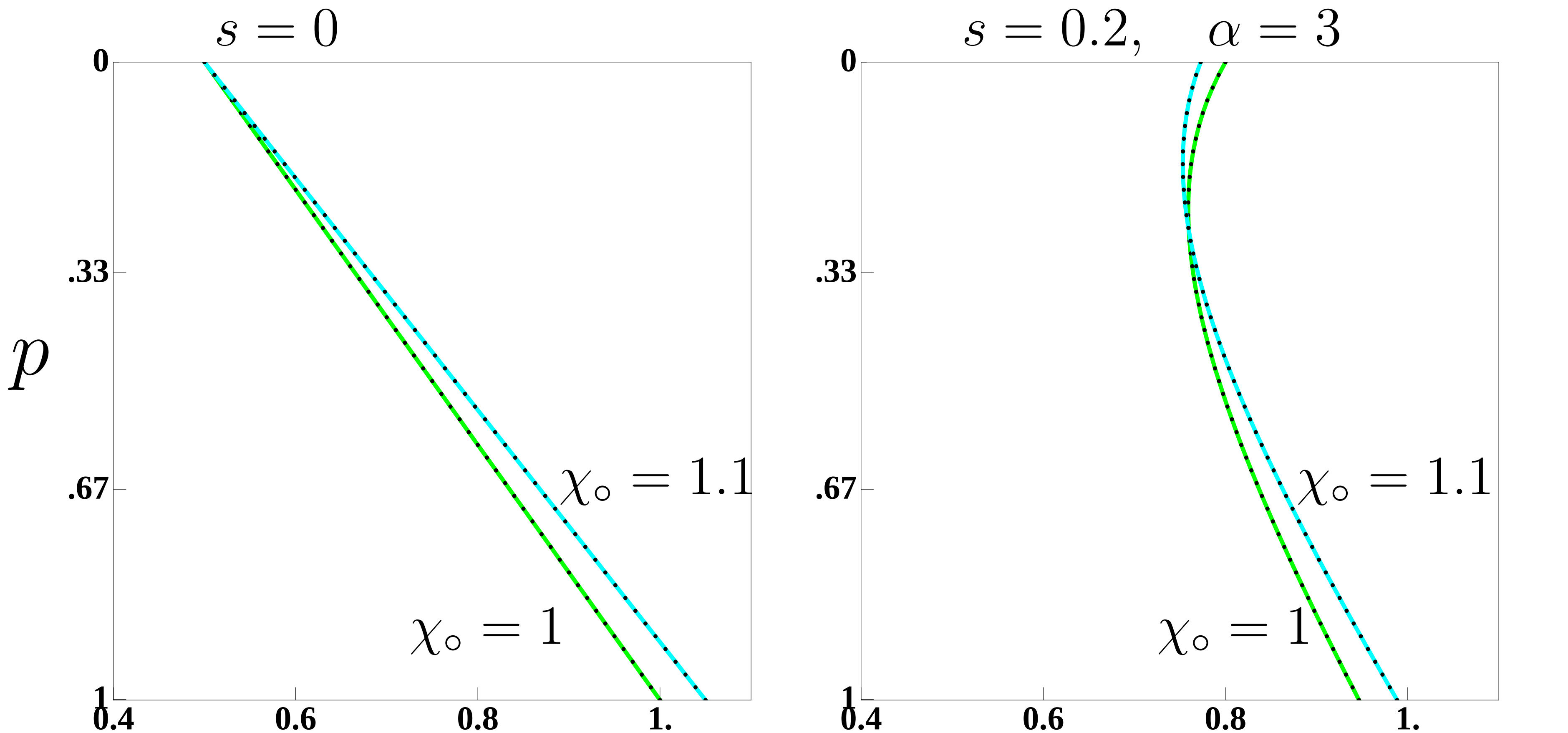
$$\sigma T^4 = F\downarrow + \frac{1}{\chi_\circ}\frac{\mathrm{d}F\downarrow}{\mathrm{d}p}$$

$$F\uparrow = F\downarrow + Q$$



Gray-atmosphere solution. Left: no solar absorption. Right: with solar absorption. All flux values normalized by $F_\circ$.

As $\chi_\circ$ increases, the surface air warms but the stratosphere cools:



σT^4 (normalized by $F_\circ$) for $\chi_\circ = 1$ and $\chi_\circ = 1.1$

Why? Recall the zero net heating condition:

$$-\frac{\mathrm{d}Q}{\mathrm{d}p} + \frac{\mathrm{d}F\uparrow}{\mathrm{d}p} - \frac{\mathrm{d}F\downarrow}{\mathrm{d}q} = 0$$

$$-\frac{\mathrm{d}Q}{\mathrm{d}p} + \chi_\circ \left(F\downarrow + F\uparrow - 2\sigma T^4\right) = 0$$

Applied to the top of the atmosphere:

$$\alpha F_\circ s + \chi_\circ \left(F_\circ - 2\sigma T^4\right) = 0$$

$\alpha F_\circ s$ is solar heating. $\chi_\circ F_\circ$ is longwave heating. $\chi_\circ 2\sigma T^4$ is longwave cooling.

This can be rearranged as:

$$\frac{1}{F_\circ}\sigma T^4 = \frac{1}{2} + \frac{\alpha}{2\chi_\circ} s$$

As $\chi_\circ$ increases, σT^4 will decrease.

This allows longwave cooling to balance the solar heating, which is assumed invariant here.