Integrals Useful for PDFs

- **Forms with e^{-x}**

$$\int_0^\infty x^{-1/2}\, e^{-x}\, dx$$

$$\sqrt{\pi}$$

$$\int_0^\infty e^{-x}\, dx$$

$$1$$

$$\int_0^\infty x^{1/2}\, e^{-x}\, dx$$

$$\frac{\sqrt{\pi}}{2}$$

$$\int_0^\infty x^{1}\, e^{-x}\, dx$$

$$1$$

$$\int_0^\infty x^{3/2}\, e^{-x}\, dx$$

$$\frac{3\sqrt{\pi}}{4}$$

$$\int_0^\infty x^{2}\, e^{-x}\, dx$$

$$2$$

$$\int_0^\infty x^{5/2}\, e^{-x}\, dx$$

$$\frac{15\sqrt{\pi}}{8}$$

$$\int_0^\infty x^{3}\, e^{-x}\, dx$$

$$6$$

- **Forms with e^{-x^2}**

$$\int_0^\infty x^{-1/2}\, e^{-x^2}\, dx$$

$$2\,\text{Gamma}\left[\frac{5}{4}\right]$$

$$\int_0^\infty e^{-x^2}\,dx$$

$$\frac{\sqrt{\pi}}{2}$$

$$\int_0^\infty x^{1/2}\,e^{-x^2}\,dx$$

$$\frac{1}{2}\,\text{Gamma}\left[\frac{3}{4}\right]$$

$$\int_0^\infty x^{1}\,e^{-x^2}\,dx$$

$$\frac{1}{2}$$

$$\int_0^\infty x^{3/2}\,e^{-x^2}\,dx$$

$$\frac{1}{2}\,\text{Gamma}\left[\frac{5}{4}\right]$$

$$\int_0^\infty x^{2}\,e^{-x^2}\,dx$$

$$\frac{\sqrt{\pi}}{4}$$

$$\int_0^\infty x^{5/2}\,e^{-x^2}\,dx$$

$$\frac{1}{2}\,\text{Gamma}\left[\frac{7}{4}\right]$$

$$\int_0^\infty x^{3}\,e^{-x^2}\,dx$$

$$\frac{1}{2}$$

```
Gamma[3 / 4.]
```

```
1.22542
```

```
Gamma[5 / 4.]
```

```
0.906402
```

```
Gamma[7 / 4.]
```

```
0.919063
```

■ Forms useful for the Planck function

$$\int_0^\infty \frac{x^4}{e^x - 1}\, dx$$

```
24 Zeta[5]
```

```
24 Zeta[5] // N
```

```
24.8863
```

$$\int_0^\infty \frac{x^3}{e^x - 1}\, dx$$

$$\frac{\pi^4}{15}$$

$$\int_0^\infty \frac{x^2}{e^x - 1}\, dx$$

```
2 Zeta[3]
```

```
2 Zeta[3] // N
```

```
2.40411
```

$$\int_0^\infty \frac{x}{e^x - 1}\, dx$$

$$\frac{\pi^2}{6}$$