METR 5223: Atmospheric Radiation

Line shape and average transmittance over ν

Lecture for Spring 2009, v0.3

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Schwarzschild equation:

$$\frac{dL_\nu}{ds} = -k_\nu \rho_a L_\nu + k_\nu \rho_a B_\nu$$

Define *optical path* χ_ν:

$$\chi_\nu(s) = \int_{s_0}^{s} k_\nu(s')\rho_a(s')ds'$$

or sometimes

$$\chi_\nu(s) = k_\nu \int_{s_0}^{s} \rho_a(s')ds' \equiv k_\nu u$$

So

$$\frac{dL_\nu}{d\chi_\nu} = -L_\nu + B_\nu$$

The solution for an isothermal layer of temperature T :

$$L_\nu(\chi_\nu) = L_\nu(0)e^{-\chi_\nu} + B_\nu(T)\left(1 - e^{-\chi_\nu}\right)$$

or

$$L_\nu(\chi_\nu) = L_\nu(0)\tau_\nu + B_\nu(T)\left(1 - \tau_\nu\right)$$

Consider

$$k_\nu = S f(\nu)$$

where S is the *line strength* and $\int f(\nu) d\nu = 1$. One example is the Lorentz line:

$$f(\nu) = \frac{\gamma}{\pi} \frac{1}{\gamma^2 + (\nu - \nu_0)^2}$$

or

$$f(\nu) = \frac{1}{\gamma\pi} \frac{1}{1 + \left(\frac{\nu-\nu_0}{\gamma}\right)^2}$$

Thus the Lorentz line can be written:

$$k_\nu = S\frac{\gamma}{\pi}\frac{1}{\gamma^2 + (\nu - \nu_0)^2}$$

$$k_\nu = \frac{S}{\pi\gamma}\frac{\gamma^2}{\gamma^2 + (\nu - \nu_0)^2}$$

We see that:

$$k_{\max} = \frac{S}{\pi\gamma} \qquad \text{or} \qquad S = k_{\max}\pi\gamma$$

For $\chi_\nu << 1$,

$$1 - e^{-\chi_\nu} = \chi_\nu = k_\nu u$$

$$B_\nu \left(1 - e^{-\chi_\nu}\right) = B_\nu k_\nu u$$

For $f(\nu)$ concentrated near ν_0,

$$\int\limits_{\nu=\nu_1}^{\nu=\nu_2} B_\nu(\nu, T) k_\nu u d\nu = B_\nu(\nu_0, T) S u \int\limits_{\nu=\nu_1}^{\nu=\nu_2} f d\nu = B_\nu(\nu_0, T) W$$

with equivalent width $W \equiv Su$.

A narrow example line:

$$S_e = 1.0 \times 10^{11} \text{ Hz m}^2 \text{ kg}^{-1}$$

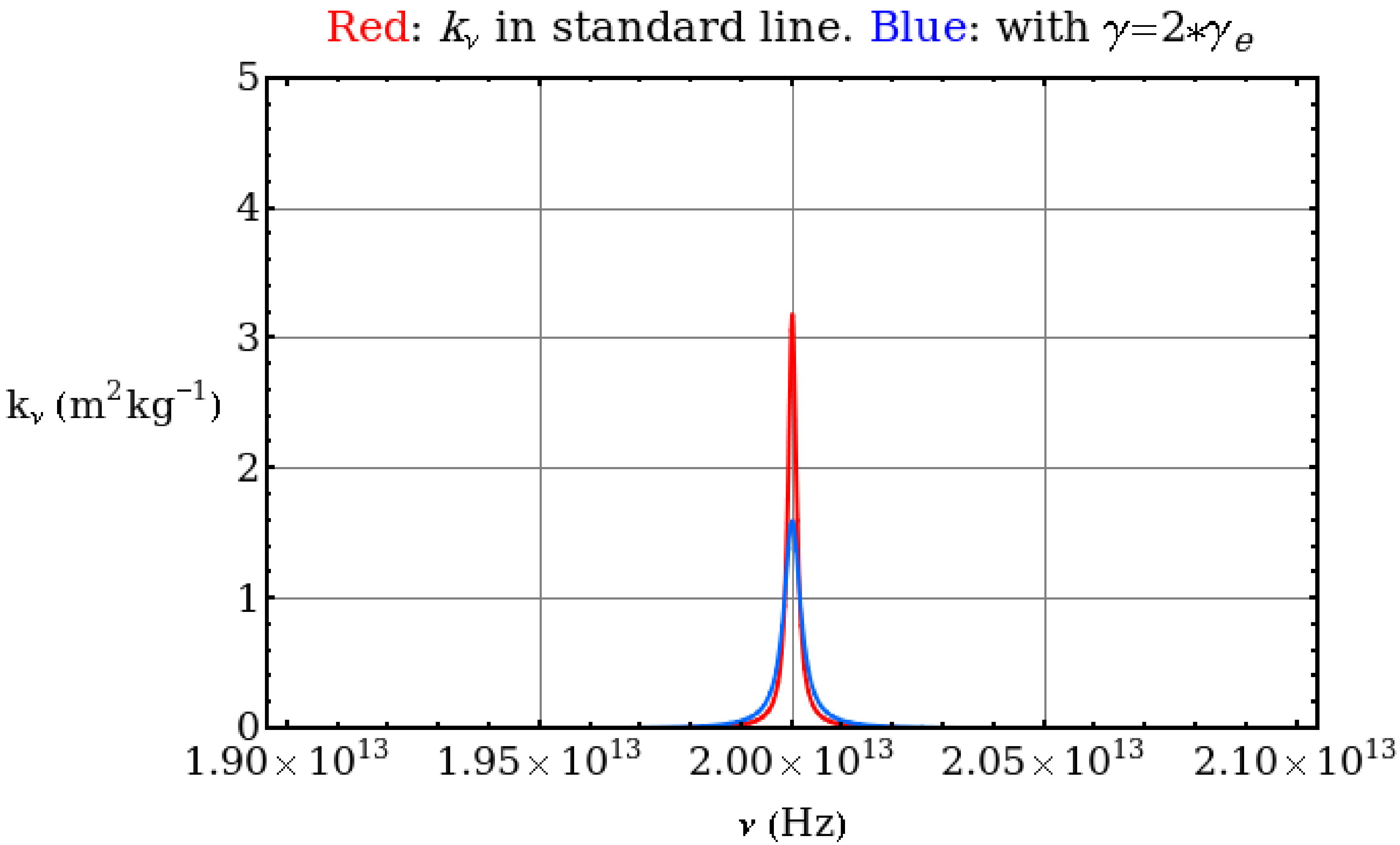
$$\nu_{0e} = 2.0 \times 10^{13} \text{ Hz}$$

$$\gamma_e = .001 \times 10^{13} \text{ Hz}$$

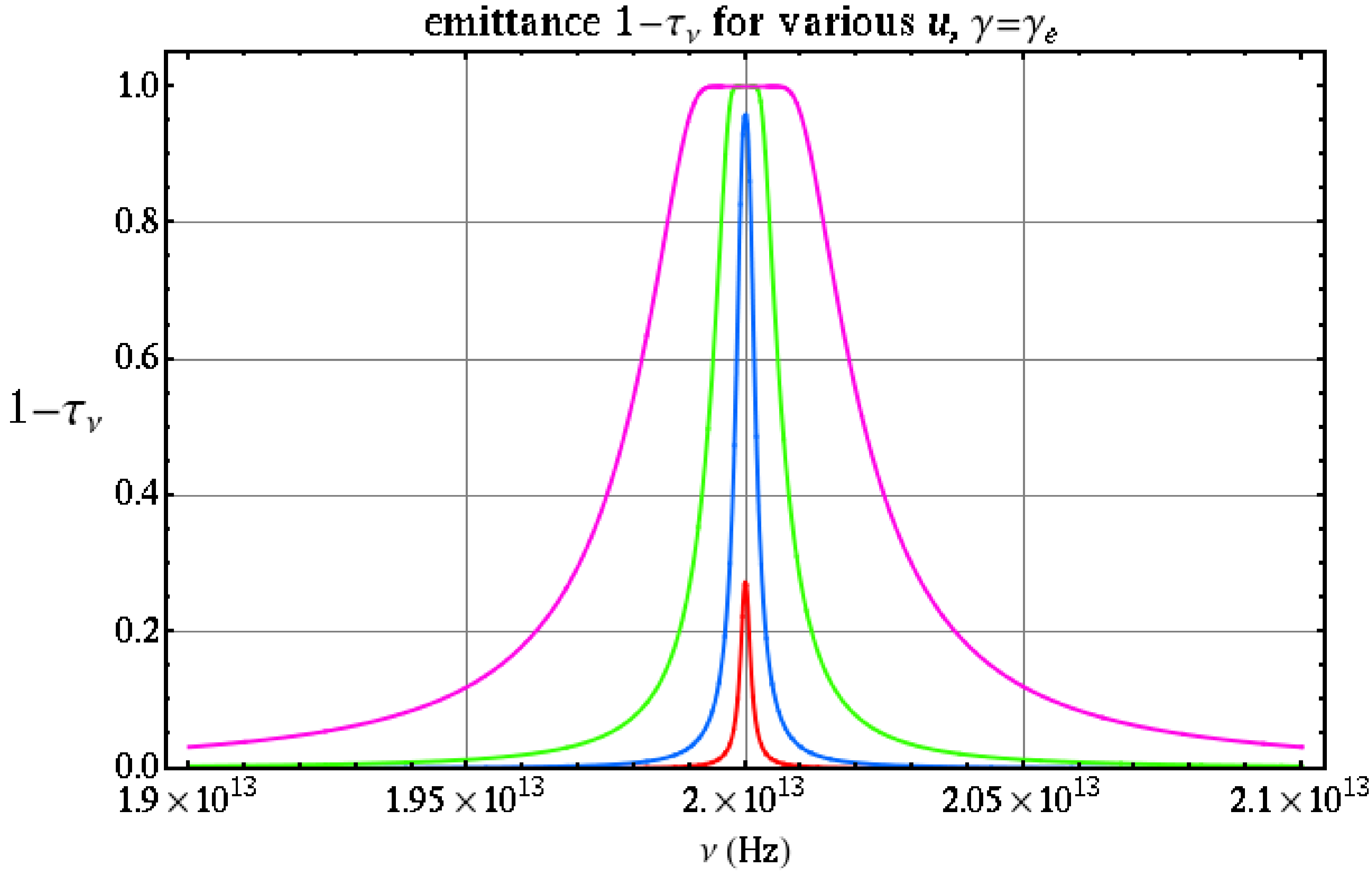
A standard *mass path* to apply:

$$u_e = 0.1 \text{ kg m}^{-2}$$

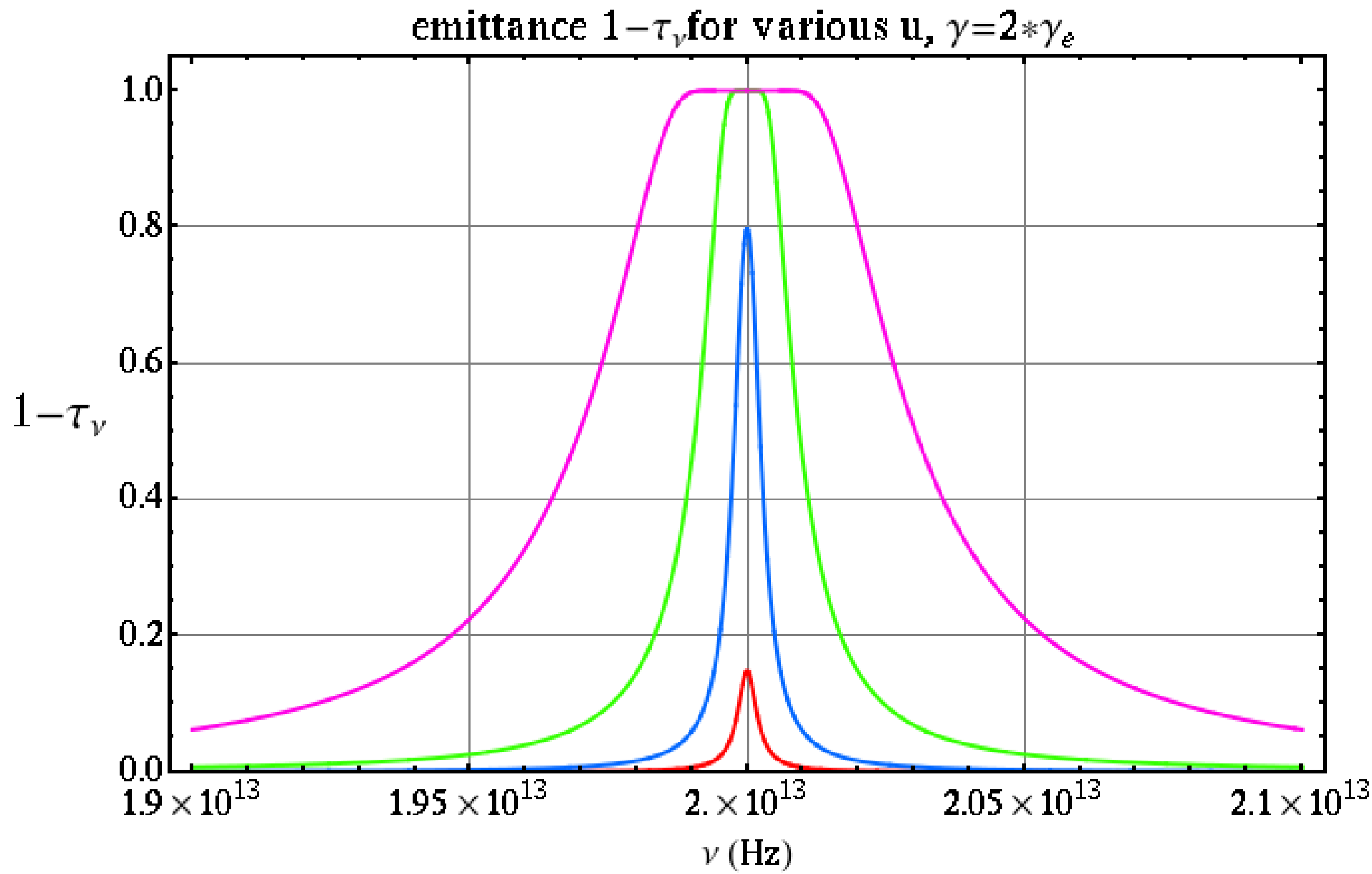
(We will explore with u in multiples of u_e)



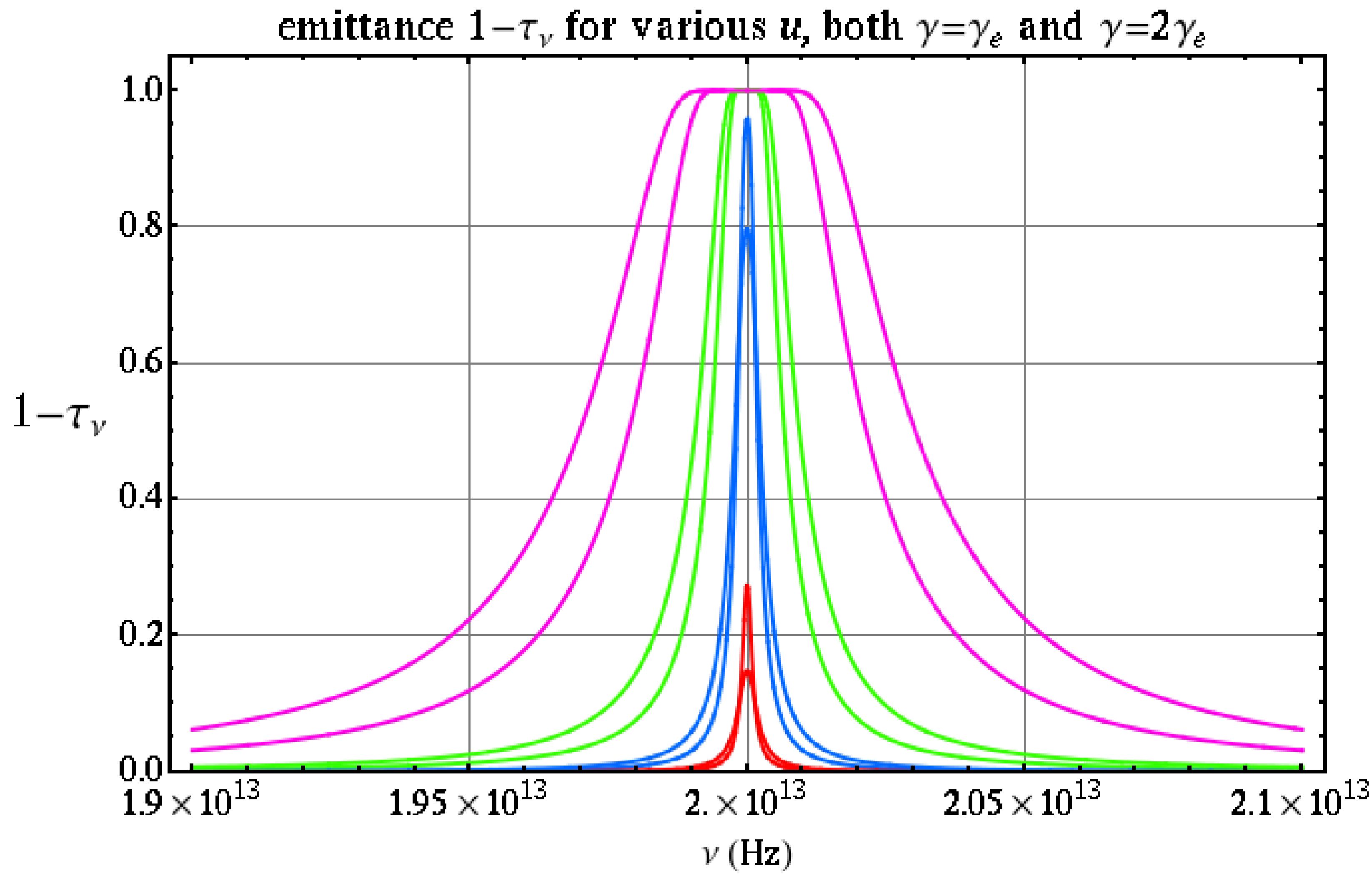
Red: k_ν in standard line. Blue: with $\gamma=2*\gamma_e$
5
4
3
2
1
0
k_ν ($m^2 kg^{-1}$)
1.90×10^{13}
1.95×10^{13}
2.00×10^{13}
2.05×10^{13}
2.10×10^{13}
ν (Hz)



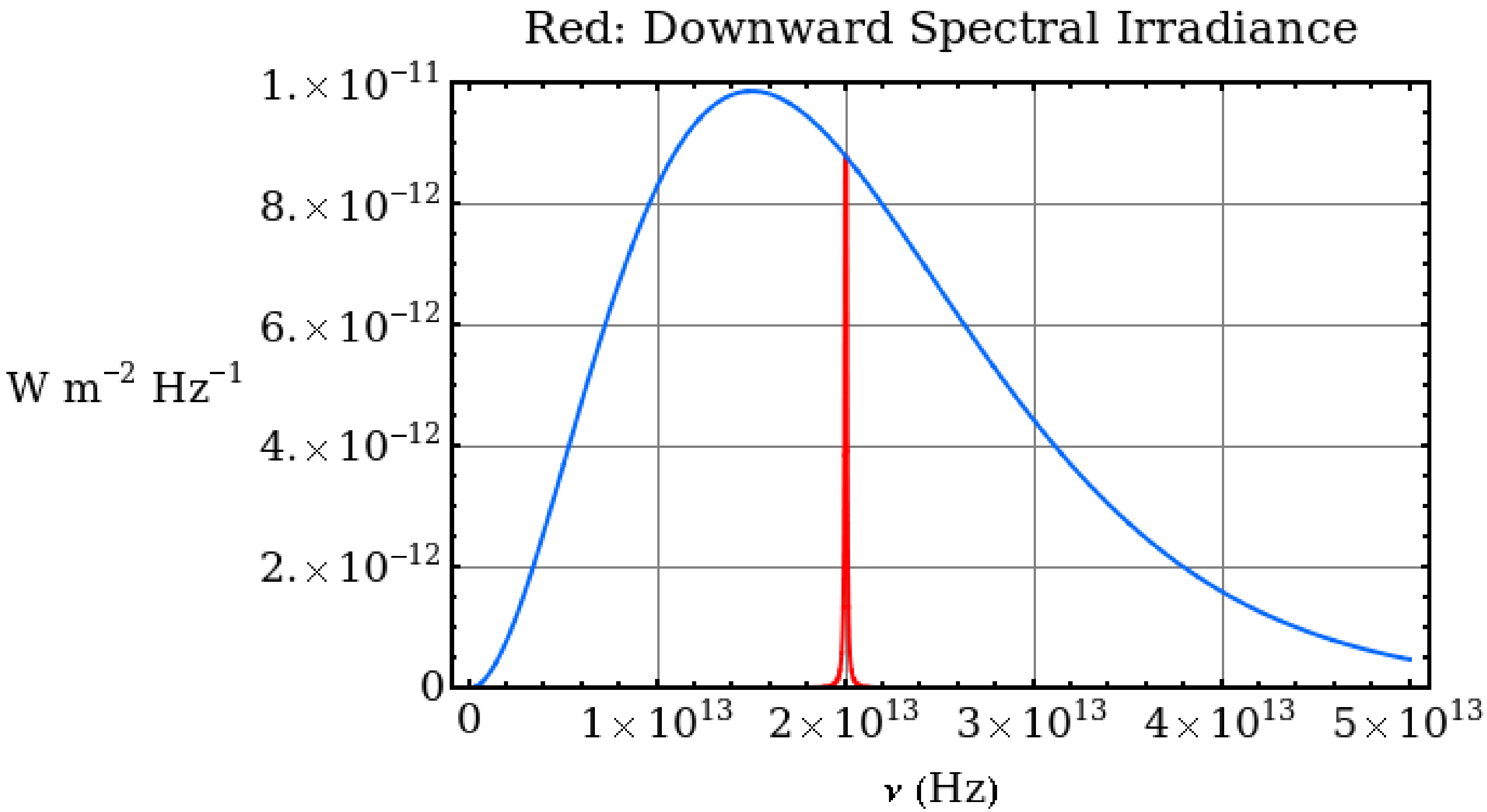
red: $u = u_e$, blue: $u = 10u_e$, green: $u = 100u_e$, purple: $u = 1000u_e$



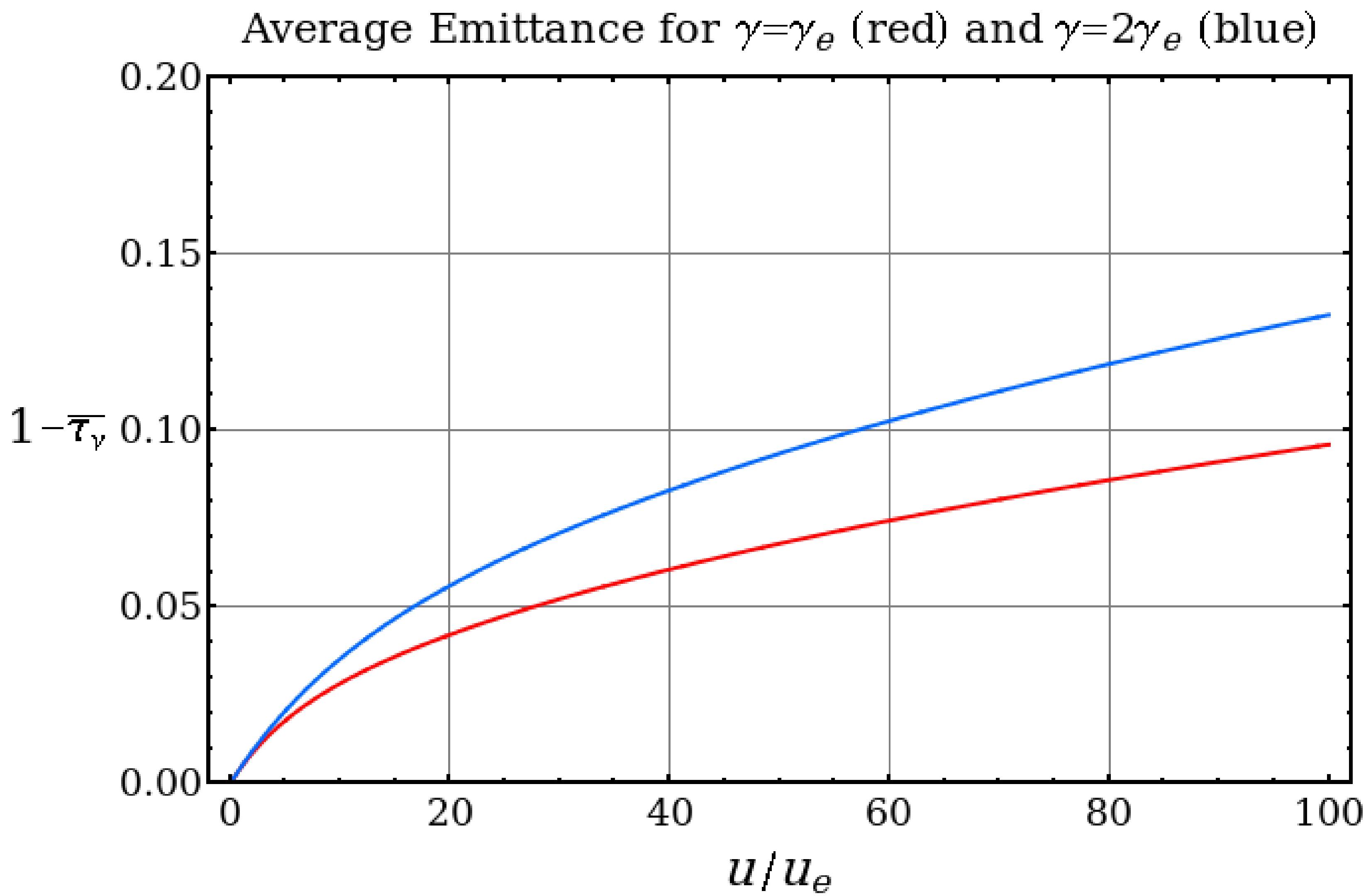
red: $u = u_e$, blue: $u = 10u_e$, green: $u = 100u_e$, purple: $u = 1000u_e$



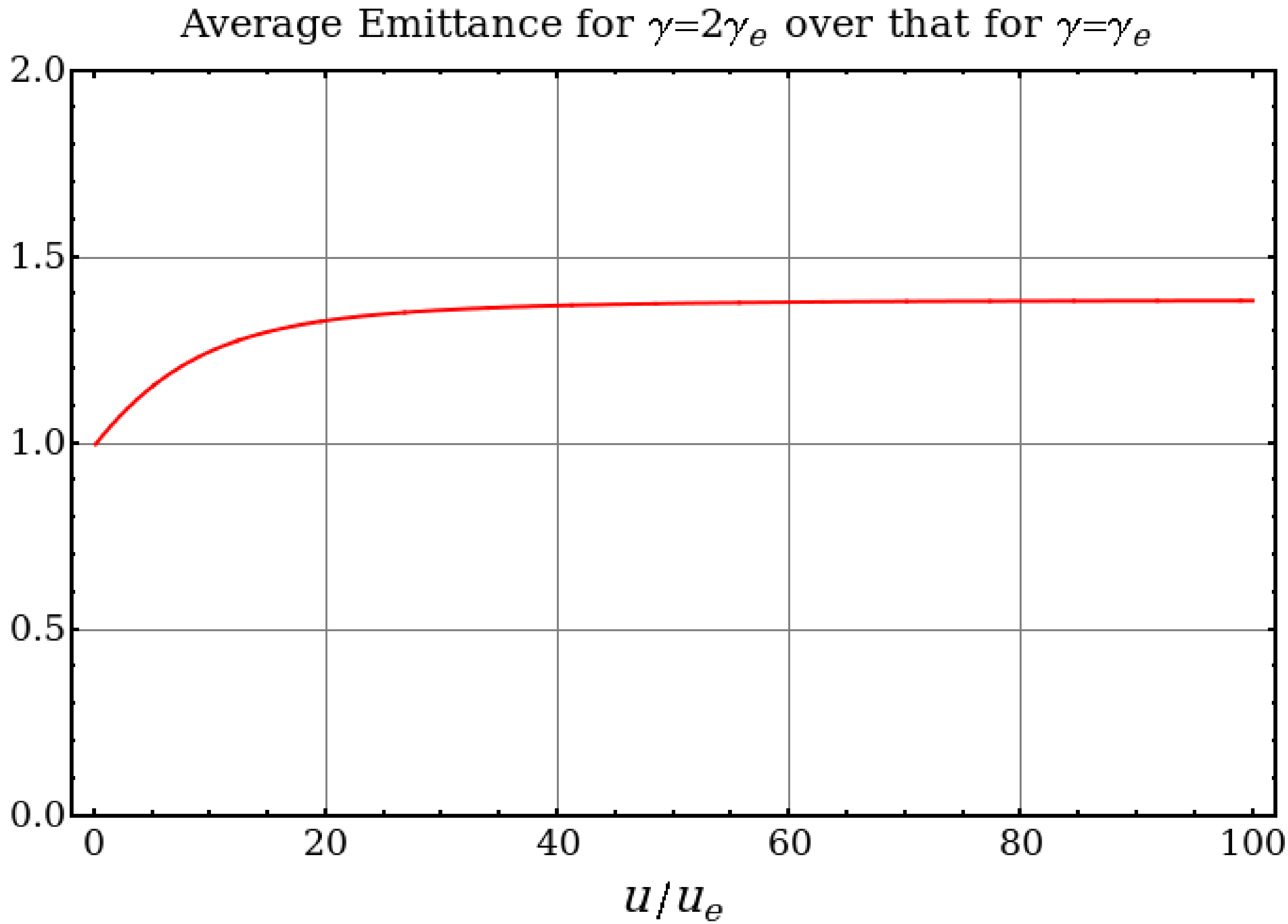
red: $u = u_e$, blue: $u = 10u_e$, green: $u = 100u_e$, purple: $u = 1000u_e$



red: downward spectral irradiance from 255 K atmosphere, using $u = 100u_e$



Is average emittance proportional to $\sqrt{\gamma}$ and $\sqrt{u}$?



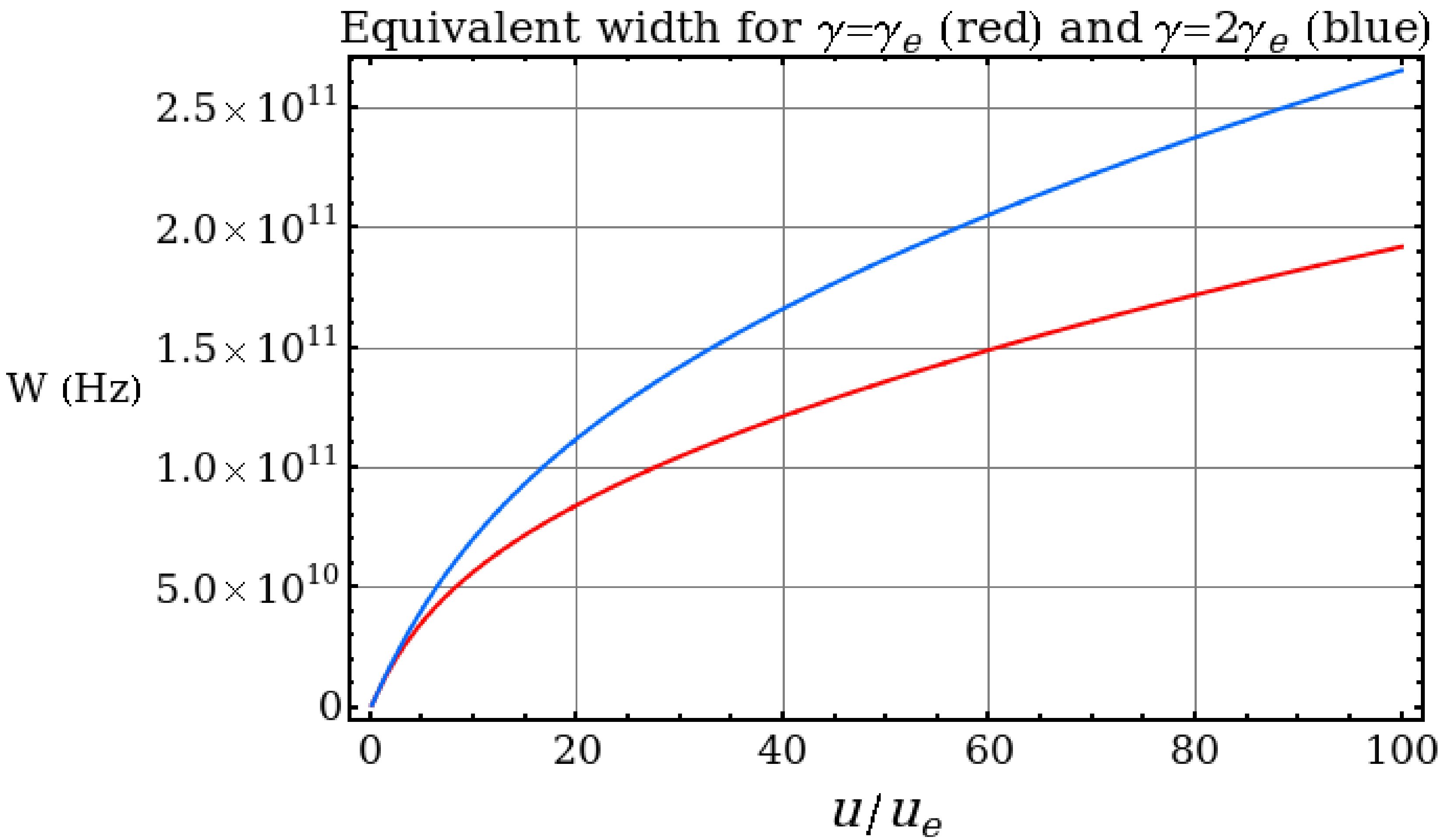
Average Emittance for γ=2γ_e over that for γ=γ_e
2.0
1.5
1.0
0.5
0.0
0
20
40
60
80
100
u/u_e

Equivalent Width vs. Average Emittance

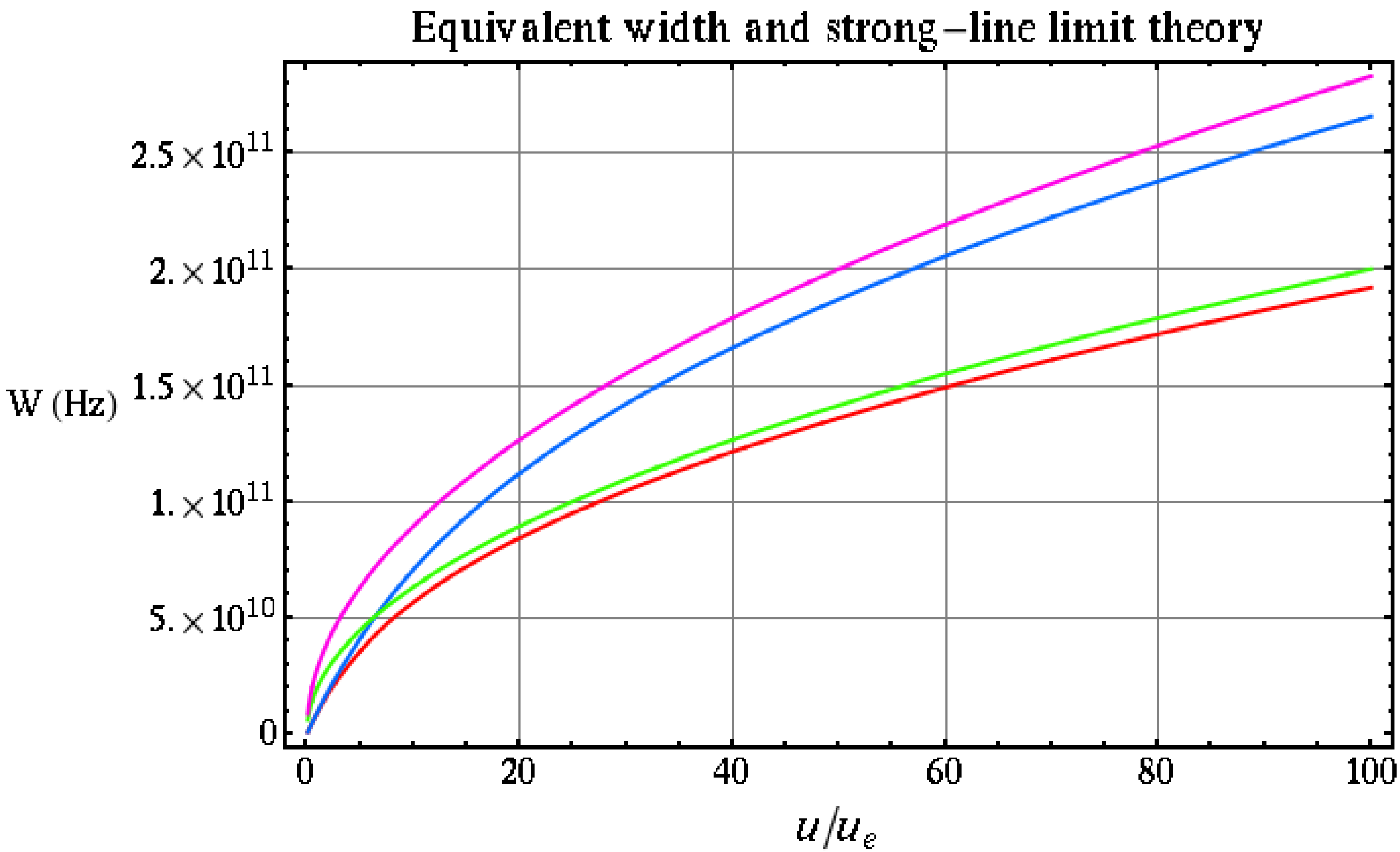
$$\int_{\nu_1}^{\nu_2} B_\nu(\nu, T)\,(1 - \tau_\nu)\,d\nu = B_\nu(\nu_0, T)\int_{\nu_1}^{\nu_2} (1 - \tau_\nu)\,d\nu$$

$$= B_\nu(\nu_0, T)W$$

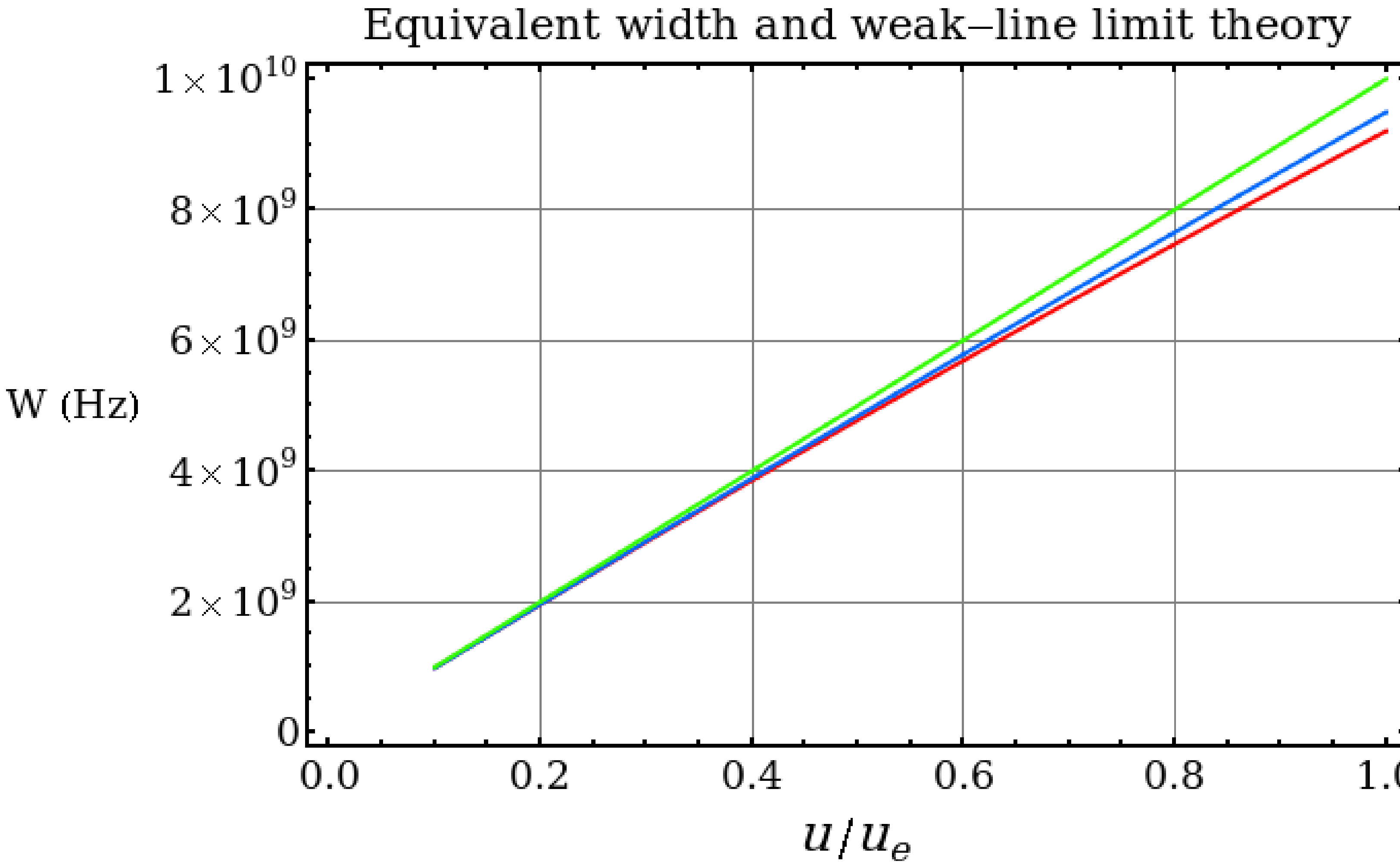
$$= B_\nu(\nu_0, T)(\nu_2 - \nu_1)(1 - \overline{\tau}_\nu)$$



Equivalent width for $\gamma=\gamma_e$ (red) and $\gamma=2\gamma_e$ (blue)
W (Hz)
2.5×10^{11}
2.0×10^{11}
1.5×10^{11}
1.0×10^{11}
5.0×10^{10}
0
0
20
40
60
80
100
u/u_e



Strong line theory: $W = 2\sqrt{Su\gamma}$



Weak line theory: $W = Su$

Explanation for strong line theory: $W = 2\sqrt{Su\gamma}$

Let $x \equiv \nu - \nu_0$.

$$\chi_\nu = k_\nu u = S\frac{\gamma}{\pi}\frac{1}{\gamma^2 + x^2}u$$

For what x is $\chi_\nu = 1$?

$$1 = S\frac{\gamma}{\pi}\frac{1}{x^2}u$$

$$x = \frac{1}{\sqrt{\pi}}\sqrt{Su\gamma} \approx W$$