METR 5223: Atmospheric Radiation

1:

Line shape and average transmittance over ν

Lecture for Spring 2009, v0.3

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2:

Schwarzschild equation:

$$\frac{dL_\nu}{ds} = -k_\nu \rho_a L_\nu + k_\nu \rho_a B_\nu$$

Define *optical path* χ_ν:

$$\chi_\nu(s) = \int_{s_0}^{s} k_\nu(s')\rho_a(s')ds'$$

or sometimes

$$\chi_\nu(s) = k_\nu \int_{s_0}^{s} \rho_a(s')ds' \equiv k_\nu u$$

3:

So

$$\frac{dL_\nu}{d\chi_\nu} = -L_\nu + B_\nu$$

4:

The solution for an isothermal layer of temperature T :

$$L_\nu(\chi_\nu) = L_\nu(0)e^{-\chi_\nu} + B_\nu(T)\left(1 - e^{-\chi_\nu}\right)$$

or

$$L_\nu(\chi_\nu) = L_\nu(0)\tau_\nu + B_\nu(T)\left(1 - \tau_\nu\right)$$

Consider

$$k_\nu = Sf(\nu)$$

where S is the *line strength* and $\int f(\nu)d\nu = 1$. One example is the Lorentz line:

5:

$$f(\nu) = \frac{\gamma}{\pi}\frac{1}{\gamma^2 + (\nu - \nu_0)^2}$$

or

$$f(\nu) = \frac{1}{\gamma\pi}\frac{1}{1 + \left(\frac{\nu-\nu_0}{\gamma}\right)^2}$$

Thus the Lorentz line can be written:

$$k_\nu = S\frac{\gamma}{\pi}\frac{1}{\gamma^2 + (\nu - \nu_0)^2}$$

6:

$$k_\nu = \frac{S}{\pi\gamma}\frac{\gamma^2}{\gamma^2 + (\nu - \nu_0)^2}$$

We see that:

$$k_{\max} = \frac{S}{\pi\gamma} \qquad \text{or} \qquad S = k_{\max}\pi\gamma$$

For $\chi_\nu << 1$,

$$1 - e^{-\chi_\nu} = \chi_\nu = k_\nu u$$

$$B_\nu\left(1 - e^{-\chi_\nu}\right) = B_\nu k_\nu u$$

7: For $f(\nu)$ concentrated near ν_0,

$$\int_{\nu=\nu_1}^{\nu=\nu_2} B_\nu(\nu, T)k_\nu u d\nu = B_\nu(\nu_0, T)Su\int_{\nu=\nu_1}^{\nu=\nu_2} f d\nu = B_\nu(\nu_0, T)W$$

with equivalent width $W \equiv Su$.

A narrow example line:

$$S_e = 1.0 \times 10^{11}\ \text{Hz m}^2\ \text{kg}^{-1}$$

$$\nu_{0e} = 2.0 \times 10^{13}\ \text{Hz}$$

8:

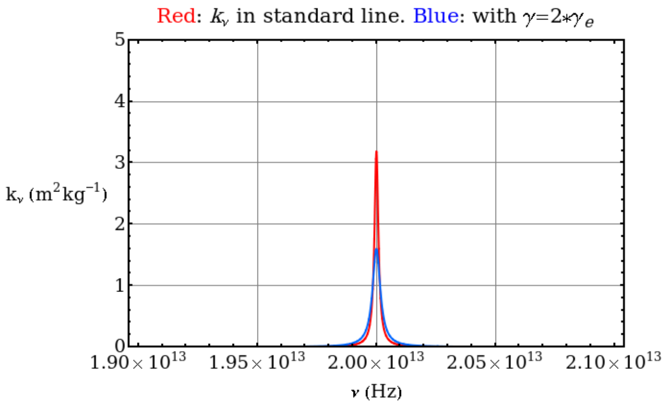
$$\gamma_e = .001 \times 10^{13}\ \text{Hz}$$

A standard *mass path* to apply:

$$u_e = 0.1\ \text{kg m}^{-2}$$

(We will explore with u in multiples of u_e)

9:



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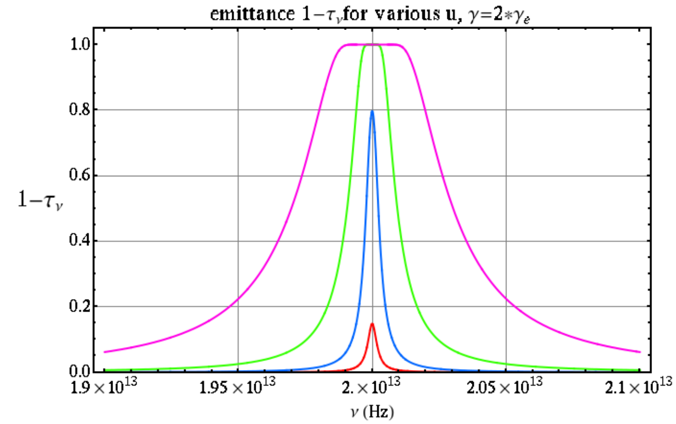
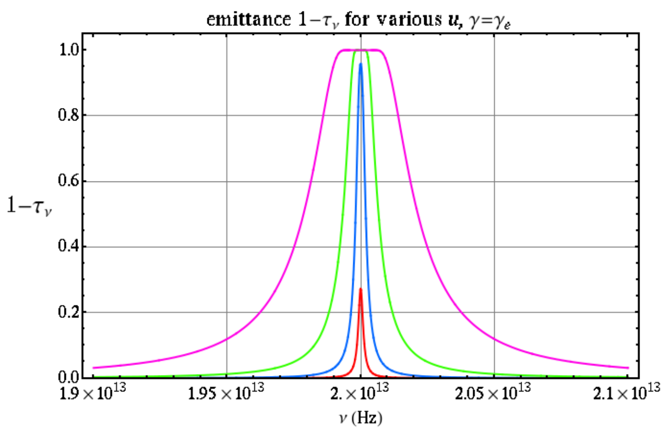


red: $u = u_e$, blue: $u = 10u_e$, green: $u = 100u_e$, purple: $u = 1000u_e$

10:



red: $u = u_e$, blue: $u = 10u_e$, green: $u = 100u_e$, purple: $u = 1000u_e$

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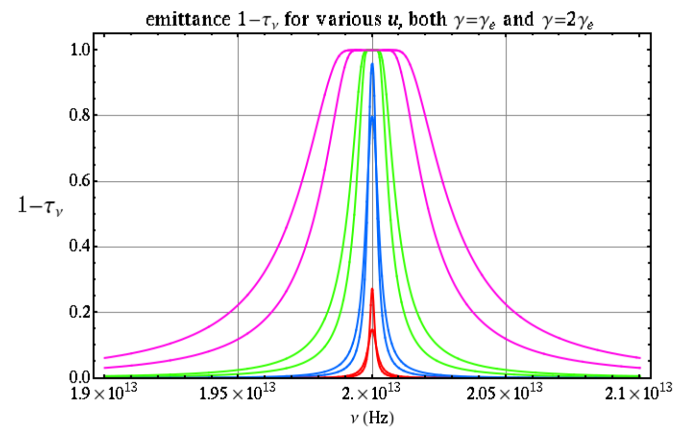
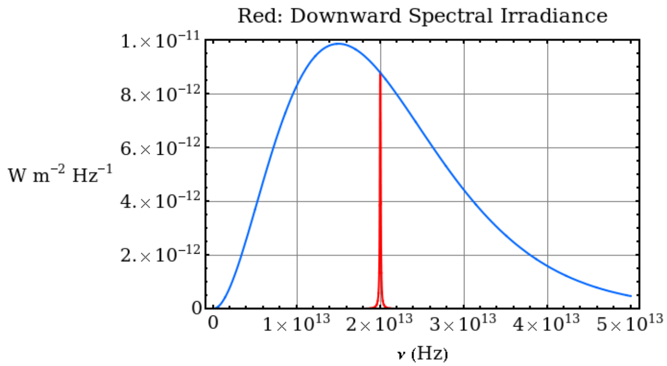


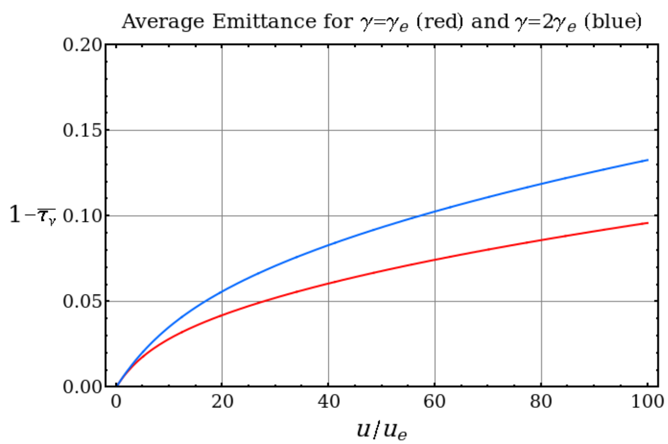
red: $u = u_e$, blue: $u = 10u_e$, green: $u = 100u_e$, purple: $u = 1000u_e$

13:



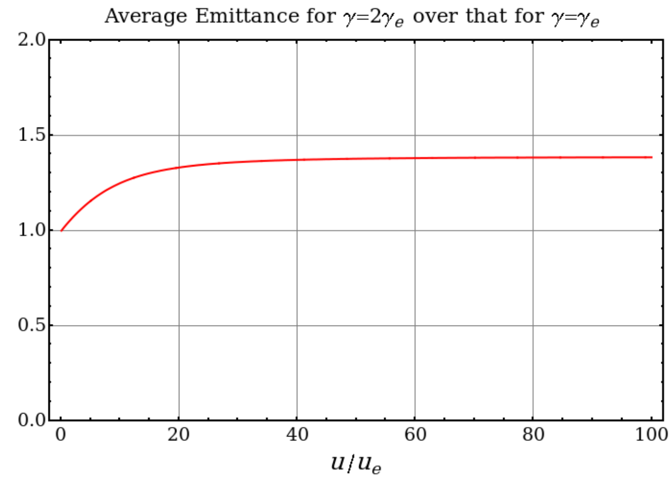
red: downward spectral irradiance from 255 K atmosphere, using $u = 100u_e$

14:



Is average emittance proportional to $\sqrt{\gamma}$ and $\sqrt{u}$?

15:

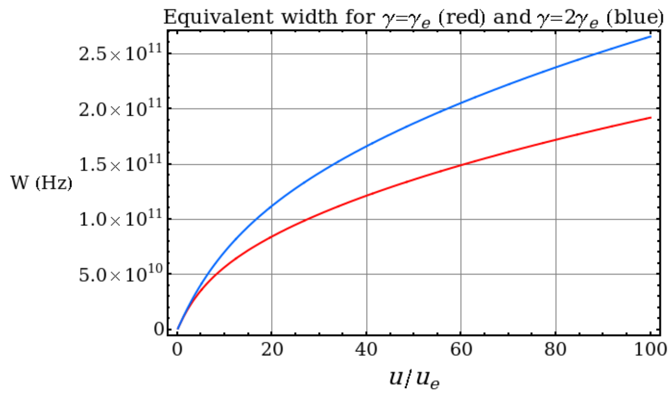


16:

Equivalent Width vs. Average Emittance

$$\begin{aligned}
\int_{\nu_1}^{\nu_2} B_\nu(\nu,T)\,(1-\tau_\nu)\,d\nu &= B_\nu(\nu_0,T)\int_{\nu_1}^{\nu_2}(1-\tau_\nu)\,d\nu \\
&= B_\nu(\nu_0,T)W \\
&= B_\nu(\nu_0,T)(\nu_2-\nu_1)(1-\overline{\tau}_\nu)
\end{aligned}$$

17:



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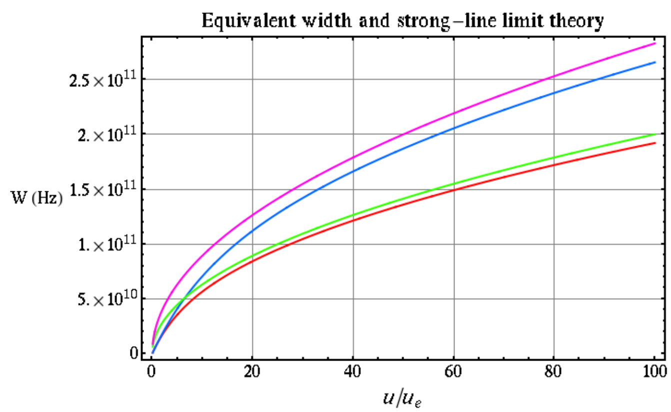
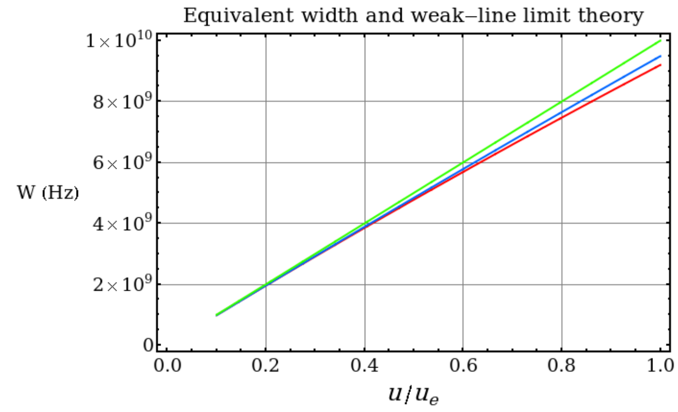


Strong line theory: $W = 2\sqrt{Su\gamma}$

19:



Weak line theory: $W = Su$

20: Explanation for strong line theory: $W = 2\sqrt{Su\gamma}$

Let $x \equiv \nu - \nu_0$.

$$\chi_\nu = k_\nu u = S\frac{\gamma}{\pi}\frac{1}{\gamma^2 + x^2}u$$

For what x is $\chi_\nu = 1$?

$$1 = S\frac{\gamma}{\pi}\frac{1}{x^2}u$$

$$x = \frac{1}{\sqrt{\pi}}\sqrt{Su\gamma} \approx W$$