

# Climate Sensitivity and Stability in a Simple Model

1:

Lecture for Spring 2009  
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Recall in the slab atmosphere model the irradiance or flux density out of the top of the atmosphere is:

$$F \uparrow = (1 - \epsilon)\sigma T_s^4 + \epsilon\sigma T_a^4$$

Recall we found that when the atmosphere is in equilibrium with the surface:

2: 
$$T_a^4 = \frac{1}{2}T_s^4$$

This equilibrium is quick, compared to the oceans coming into equilibrium with a change in solar radiation.

Assuming the atmosphere is in equilibrium with the oceans:

$$F \uparrow = \left(1 - \frac{\epsilon}{2}\right)\sigma T_s^4$$

For an “aqua-planet” with a well-mixed ocean and a slab atmosphere:

$$C \frac{dT_s}{dt} = \frac{S_0}{4}(1 - \alpha_p) - \left(1 - \frac{\epsilon}{2}\right)\sigma T_s^4$$

3: We simplify notation with  $T_s = T$  and  $(1 - \alpha_p) = B$  (think  $B$  for **blackness**) and  $R = \left(1 - \frac{\epsilon}{2}\right)$  (the greenhouse **reduction** factor) and  $s = \frac{S_0}{4}$ :

$$C \frac{dT}{dt} = sB - RT^4$$

In equilibrium:

$$RT^4 = sB$$

Use calculus to examine the sensitivity of the equilibrium solutions for  $T$ .

$$\ln(R\sigma T^4) = \ln(sB)$$

4: 
$$\ln(R) + \ln(\sigma) + 4\ln(T) = \ln(s) + \ln(B)$$

$$\frac{dR}{R} + 4\frac{dT}{T} = \frac{ds}{s} + \frac{dB}{B}$$

Recall the familiar case with  $B$  and  $R$  constant:

$$\frac{dT}{T} = \frac{1}{4} \frac{ds}{s}$$

Now investigate feedback. Suppose  $B = B(T)$ , but the changes in  $R$  and  $S$  are imposed, and independent of  $T$ :

$$\frac{dR}{R} + 4 \frac{dT}{T} = \frac{ds}{s} + \frac{1}{B} \frac{dB}{dT} dT$$

$$4 \frac{dT}{T} - \frac{1}{B} \frac{dB}{dT} dT = \frac{dP}{P}$$

5: where  $P$  can be called the **punch**

$$P \equiv \frac{s}{R}$$

$$\frac{dT}{T} \left( 1 - \frac{1}{4} \frac{T}{B} \frac{dB}{dT} \right) = \frac{1}{4} \frac{dP}{P}$$

Increasing  $s$  or decreasing  $R$  give positive changes in  $P$ .

$$\frac{dT}{dP} = G \frac{1}{4} \frac{T}{P}$$

where the **gain**  $G$  is:

$$G \equiv \frac{1}{1-f}$$

6: and  $f$  is the **feedback** factor:

$$f \equiv \frac{1}{4} \frac{d \ln B}{d \ln T}$$

When  $f = 0$ ,  $G = 1$ . We can write:

$$\frac{dT}{dP} = G \left( \frac{dT}{dP} \right)_{\text{no-feedback}}$$

$$G = \frac{1}{1-f}$$

- 7:
- $0 < G < 1$  for  $f < 0$ . This is negative feedback.
  - $G > 1$  for  $0 < f < 1$ . This is positive feedback.
  - $G < 0$  for  $f > 1$ . Unstable equilibrium point.

Attempt to estimate  $dB/dT$  for Earth and  $f$  for Albedo feedback.

Suppose the temperature drops by 30 K. Earth becomes frozen and completely white.  $B$  goes from 0.7 to 0.

8:

$$\frac{dB}{dT} \approx \frac{\Delta B}{\Delta T} = \frac{-0.7}{-30 \text{ K}} = .023 \text{ K}^{-1}$$

$$f = \frac{1}{4} \frac{T}{B} \frac{dB}{dT} = \frac{1}{4} \frac{300 \text{ K}}{0.7} \frac{0.7}{30 \text{ K}} = 2.5$$

$f = 2.5$  is an overestimate by *at least a factor of 10* for current polar ice:

1. Ice is not perfectly reflective, esp. in the near IR.
2. High latitudes have small surface area.
3. At large zenith angle water is highly reflective.

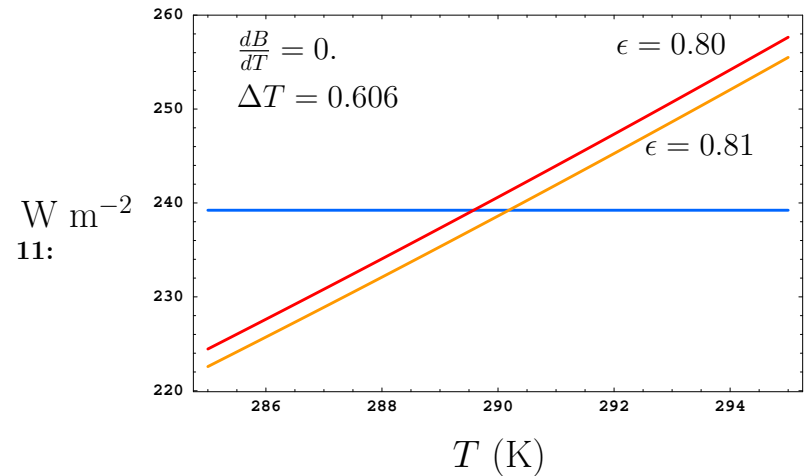
9:

For an example of moderate ice-albedo feedback we could use:

$$f = 0.25 \quad G = \frac{1}{1-f} = 1.33$$

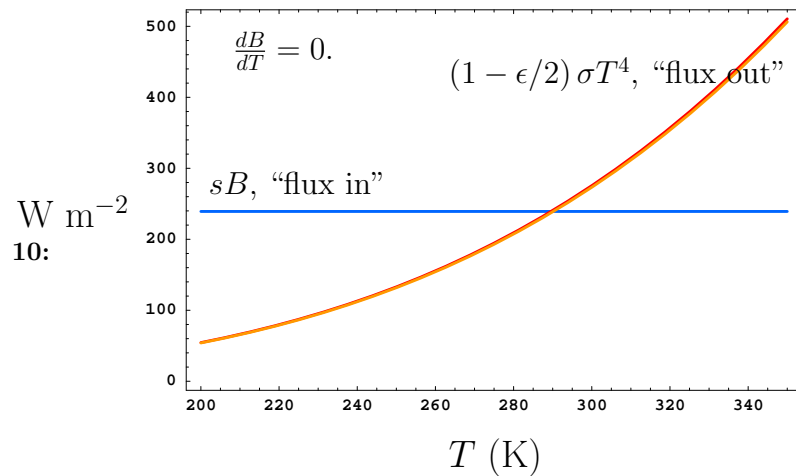
Turn now to a graphical presentation of sensitivity and feedback...

*B* constant, no feedback, zoom:



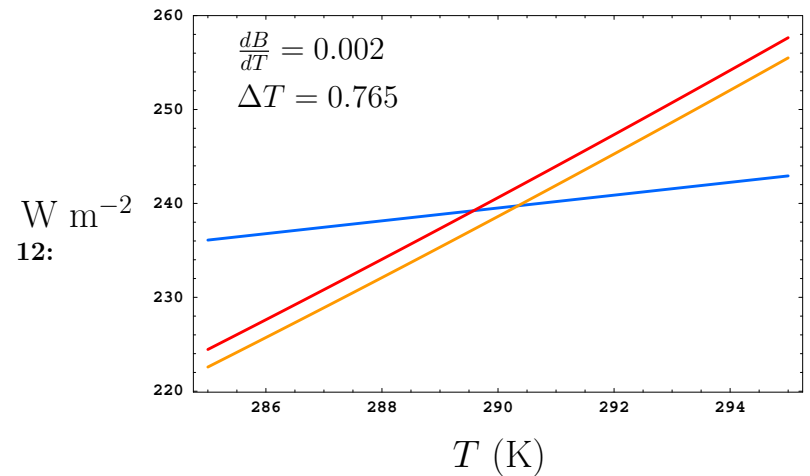
As  $\epsilon = .80 \rightarrow \epsilon = .81$ ,  $T$  increases by  $\Delta T = 0.606$

*B* constant, no feedback:



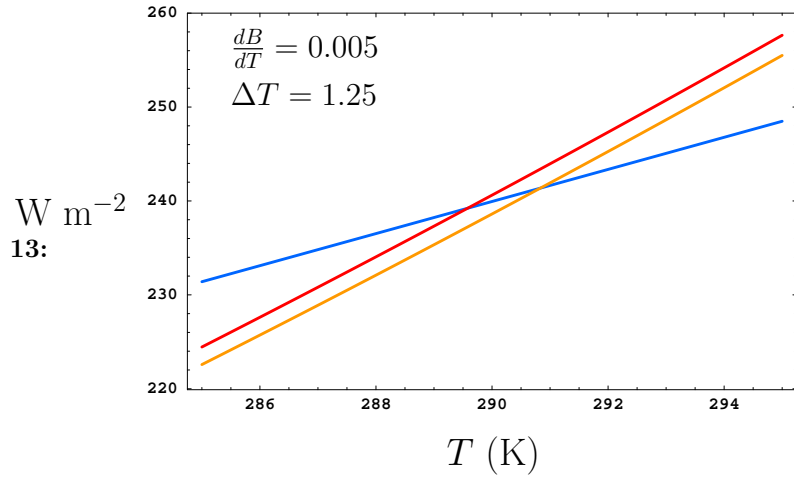
“flux out” curves for  $\epsilon = .80$  and  $\epsilon = .81$  nearly overlap

*B* increases with  $T$ , positive feedback :



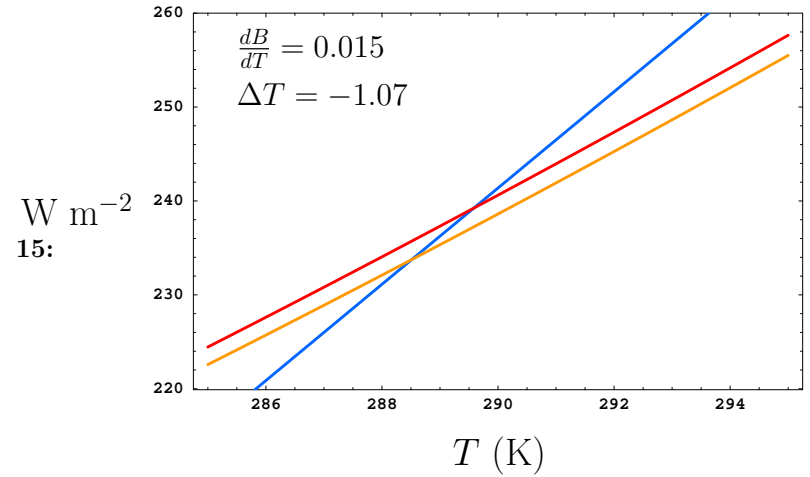
$\Delta T = 0.785$ , no-feedback was  $\Delta T = 0.606$

larger positive feedback :



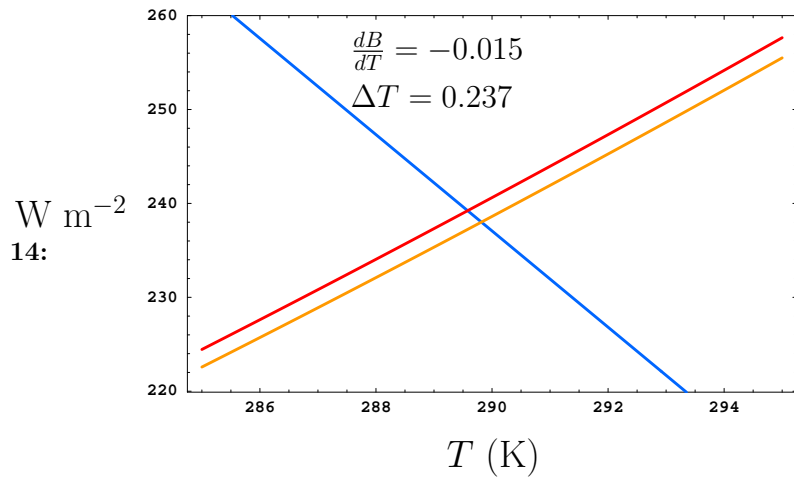
The gain  $G = 2.1$

very large positive feedback, equilibrium point unstable:



equilibrium analysis of the unstable point yields  $\Delta T < 0$

$B$  decreases with  $T$ , negative feedback:



$\Delta T > 0$ , but less than no-feedback case of  $\Delta T = 0.606$

summary:

