

METR 5223: Atmospheric Radiation

Transient response near
equilibrium, with feedback

Lecture for Spring 2009

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$$C \frac{dT}{dt} = sB - R\sigma T^4$$

$$s = \bar{s} + s'(t) \quad B = \bar{B} + B'(t) \quad T = \bar{T} + T'(t) \quad R = \bar{R} + R'(t)$$

Examine $T'(t)$ near the equilibrium solution

$$\bar{s}\bar{B} = \bar{R}\sigma\bar{T}^4$$

$$C \left(\frac{d\bar{T}}{dt} + \frac{dT'}{dt} \right) = (\bar{s} + s')(\bar{B} + B') - (\bar{R} + R')\sigma(\bar{T} + T')^4$$

With the usual linearization:

$$C \frac{dT'}{dt} = \bar{s}B' + s'\bar{B} - R'\sigma\bar{T}^4 - 4\bar{R}\sigma\bar{T}^3T'$$

Assume $B = B(T)$

$$B' = \frac{dB(\bar{T})}{dT}T' = \frac{dB}{dT}T'$$

$$C \frac{dT'}{dt} = \bar{s} \frac{dB}{dT}T' + s'\bar{B} - R'\sigma\bar{T}^4 - 4\bar{R}\sigma\bar{T}^3T'$$

$$C \frac{dT'}{dt} = \bar{s} \frac{dB}{dT}T' + s'\bar{B} - R' \frac{\bar{s}\bar{B}}{\bar{R}} - 4 \frac{\bar{s}\bar{B}}{\bar{T}}T'$$

$$\frac{C\bar{T}}{4\bar{B}\bar{s}} \frac{dT'}{dt} = \left(s' - R' \frac{\bar{s}}{\bar{R}} \right) \frac{1}{4} \frac{\bar{T}}{\bar{s}} - T' \left(1 - \frac{1}{4} \frac{\bar{T}}{\bar{B}} \frac{dB}{dT} \right)$$

$$\tau \frac{dT'}{dt} + (1 - f)T' = T'_0$$

Where

$$\tau \equiv \frac{C\bar{T}}{4\bar{B}\bar{s}} = \frac{C}{4\bar{R}\sigma\bar{T}^3}$$

$$T'_0 \equiv \left(s' - R' \frac{\bar{s}}{\bar{R}} \right) \frac{1}{4} \frac{\bar{T}}{\bar{s}}$$

$$f \equiv \frac{1}{4} \frac{\bar{T}}{\bar{B}} \frac{dB}{dT}$$

Note: for a step function in T'_0 , f has little impact initially, while $T' \ll T'_0$.

Let $G \equiv \frac{1}{1-f}$

$$\tau \frac{dT'}{dt} + (1-f)T' = T'_0 \quad \rightarrow \quad G\tau \frac{dT'}{dt} + T' = GT'_0$$

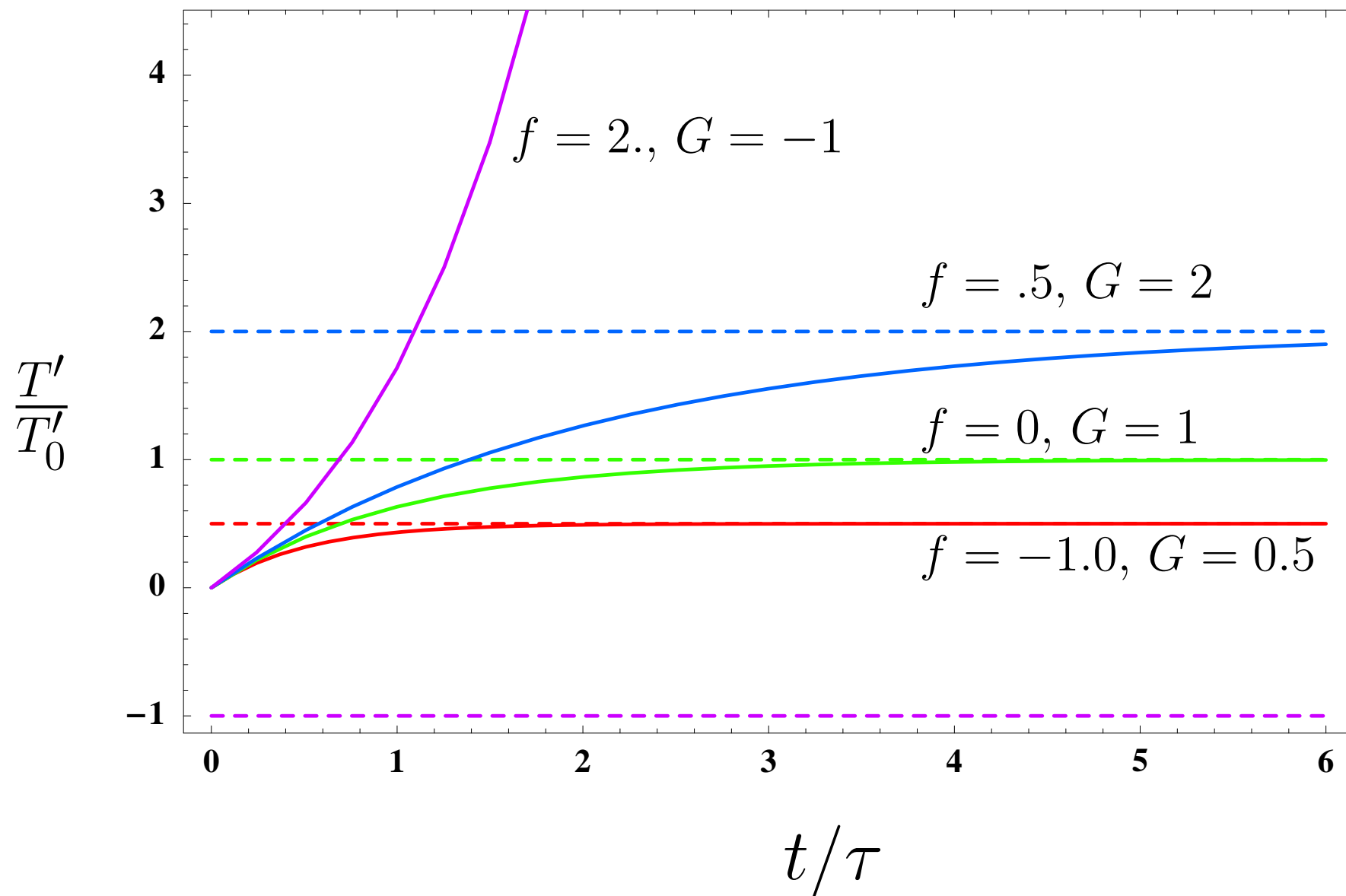
$$T'(t) = GT'_0 (1 - e^{-t/G\tau})$$

For small $\frac{t}{\tau}$: $T' = T'_0 \frac{t}{\tau}$ (independent of G).

For $G > 0$: $\lim_{t \rightarrow \infty} T'(t) = GT'_0$

For $G < 0$: $\lim_{t \rightarrow \infty} \frac{T'(t)}{T'_0} = \infty$

$$T'(t) = GT'_0 (1 - e^{-t/G\tau})$$



Suppose the climate change on Planet Z has been initiated by an ecoterrorist dumping a greenhouse gas into the atmosphere at time $t=0$. Suppose the climate experts on Planet Z are unsure of the precise value for τ for Planet Z; but the experts claim that $80 \text{ years} < \tau < 120 \text{ years}$. The experts are quite confident in their radiative transfer calculations, and they know the value of T'_0 to great precision. However, they argue wildly about the value of G , and so reach no consensus on the ultimate value of warming GT'_0 . They are resigned to waiting and observing what actually happens on Planet Z.

After 40 years, the experts have access to a measurement of T' that is risen above the noise level: $T' = 0.30T'_0$. Given the constraint they agree on for the value of τ , what is the possible range for G ?

$$T'(t) = GT'_0 (1 - e^{-t/G\tau})$$

