1:

METR 5223: Atmospheric Radiation

Transient response near equilibrium, with feedback

Lecture for Spring 2009
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2:

$$C\frac{dT}{dt} = sB - R\sigma T^4$$

$$s = \overline{s} + s'(t) \quad B = \overline{B} + B'(t) \quad T = \overline{T} + T'(t) \quad R = \overline{R} + R'(t)$$

Examine $T'(t)$ near the equilbrium solution

$$\overline{s}\,\overline{B} = \overline{R}\sigma\overline{T}^4$$

$$C\left(\frac{d\overline{T}}{dt} + \frac{dT'}{dt}\right) = (\overline{s} + s')(\overline{B} + B') - (\overline{R} + R')\sigma(\overline{T} + T')^4$$

3:

With the usual linearization:

$$C\frac{dT'}{dt} = \overline{s}B' + s'\overline{B} - R'\sigma\overline{T}^4 - 4\overline{R}\sigma\overline{T}^3 T'$$

Assume $B = B(T)$

$$B' = \frac{dB(\overline{T})}{dT}T' = \frac{dB}{dT}T'$$

$$C\frac{dT'}{dt} = \overline{s}\frac{dB}{dT}T' + s'\overline{B} - R'\sigma\overline{T}^4 - 4\overline{R}\sigma\overline{T}^3 T'$$

$$C\frac{dT'}{dt} = \overline{s}\frac{dB}{dT}T' + s'\overline{B} - R'\frac{\overline{s}\,\overline{B}}{\overline{R}} - 4\frac{\overline{s}\,\overline{B}}{\overline{T}}T'$$

4:

$$\frac{C\overline{T}}{4\overline{B}\,\overline{s}}\frac{dT'}{dt} = \left(s' - R'\frac{\overline{s}}{\overline{R}}\right)\frac{1}{4}\frac{\overline{T}}{\overline{s}} - T'\left(1 - \frac{1}{4}\frac{\overline{T}}{\overline{B}}\frac{dB}{dT}\right)$$

$$\tau\frac{dT'}{dt} + (1-f)T' = T'_0$$

Where

$$\tau \equiv \frac{C\overline{T}}{4\overline{B}\,\overline{s}} = \frac{C}{4\overline{R}\sigma\overline{T}^3}$$

$$T'_0 \equiv \left(s' - R'\frac{\overline{s}}{\overline{R}}\right)\frac{1}{4}\frac{\overline{T}}{\overline{s}}$$

$$f \equiv \frac{1}{4}\frac{\overline{T}}{\overline{B}}\frac{dB}{dT}$$

Note: for a step function in T'_0, f has little impact initially, while $T' << T'_0$.

5:

$$\text{Let} \qquad G \equiv \frac{1}{1-f}$$

$$\tau\frac{dT'}{dt} + (1-f)T' = T'_0 \qquad \rightarrow \qquad G\tau\frac{dT'}{dt} + T' = GT'_0$$

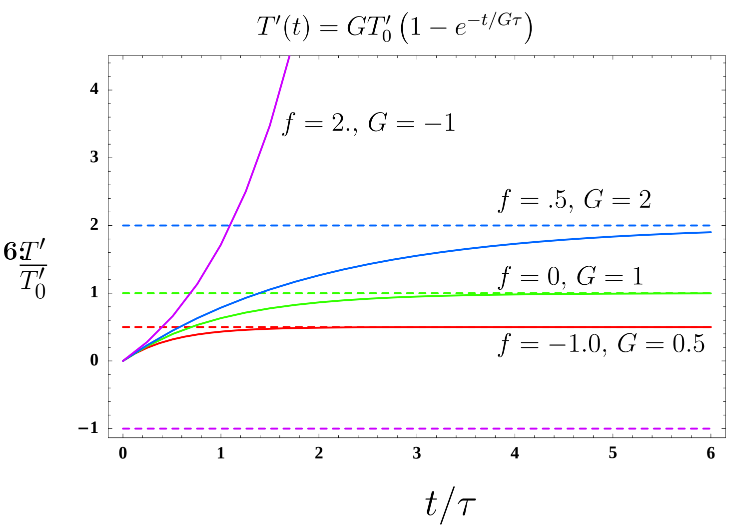
$$T'(t) = GT'_0\left(1 - e^{-t/G\tau}\right)$$

For small $\frac{t}{\tau}$: $\quad T' = T'_0\frac{t}{\tau} \quad$ (independent of G).

For $G > 0$: $\quad \lim_{t\to\infty} T'(t) = GT'_0$

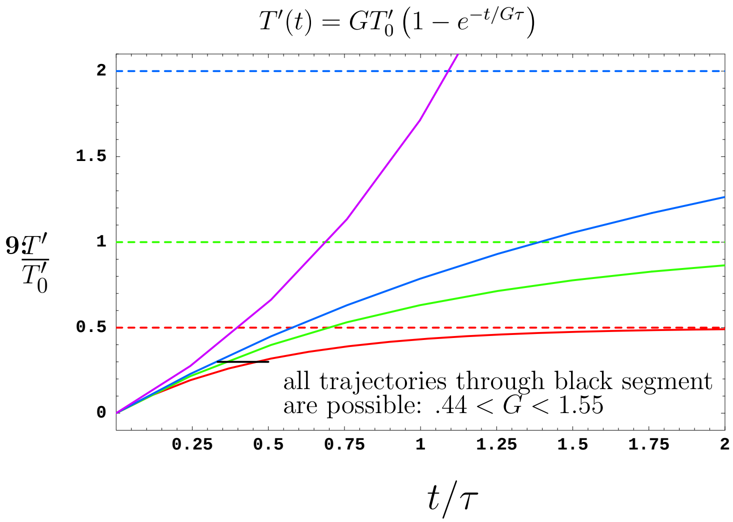
For $G < 0$: $\quad \lim_{t\to\infty} \frac{T'(t)}{T'_0} = \infty$

6:



7: Suppose the climate change on Planet Z has been initiated by an ecoterrorist dumping a greenhouse gas into the atmosphere at time t=0. Suppose the climate experts on Planet Z are unsure of the precise value for τ for Planet Z; but the experts claim that 80 years $< \tau <$ 120 years. The experts are quite confident in their radiative transfer calculations, and they know the value of T'_0 to great precision. However, they argue wildly about the value of G, and so reach no consensus on the ultimate value of warming GT'_0. They are resigned to waiting and observing what actually happens on Planet Z.

8: After 40 years, the experts have access to a measurement of T' that is risen above the noise level: $T' = 0.30T'_0$. Given the constraint they agree on for the value of τ, what is the possible range for G?



$T'(t) = G T'_0 \left(1 - e^{-t/G\tau}\right)$
2
1.5
1
0.5
0
$9\frac{T'}{T'_0}$
0.25
0.5
0.75
1
1.25
1.5
1.75
2
t/τ
all trajectories through black segment
are possible: $.44 < G < 1.55$